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# The Euro Area Interbank Market and the Liquidity Management of the Eurosystem in the Financial Crisis

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# The Euro Area Interbank Market and the Liquidity Management of the Eurosystem in the Financial Crisis

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**Abstract**

This paper develops a theoretical model which explains several stylized facts observed in the euro area interbank market after the collapse of Lehman Brothers in 2008. The model shows that if transaction costs are high, banks with a liquidity deficit will prefer to borrow liquidity from the central bank rather than from surplus banks in the interbank market. This implies that the central bank assumes an intermediary function. From a policy perspective, we argue that possible measures of the Eurosystem to reactivate the interbank market may conflict, inter alia, with monetary policy aims.

*JEL classification:* E52, E58, G01, G21

*Keywords:* Liquidity; Monetary Policy Instruments; Interbank Market; Financial Crisis

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# 1 Introduction

The worldwide financial crisis, which broke out in August 2007, triggered severe turbulence in the euro area money markets. Particularly in the aftermath of the collapse of Lehman Brothers in September 2008, several previously unseen developments could be observed. Transactions in the interbank markets fell dramatically and the interest rate for overnight interbank lending, which is usually slightly above the Eurosystem's[1] key policy rate, declined significantly below this rate. At the same time, aggregate borrowing of euro area commercial banks from the Eurosystem but also their use of the Eurosystem's deposit facility rose sharply.

In this paper, we present a theoretical model which shows that these developments in the euro area during the financial crisis can be explained by high transaction costs in the interbank market. In our model, a bank can borrow liquidity from the central bank or in the interbank market, and it can place excess liquidity at the central bank or in the interbank market. High transaction costs imply that banks with a liquidity deficit find it more attractive to cover their liquidity needs by borrowing from the central bank rather than in the interbank market. This behaviour leads to excess liquidity in the banking sector which puts downward pressure on the interbank market rate and which induces banks with a liquidity surplus to place their excess liquidity at the central bank. We thus challenge the view put forward e.g. by Trichet (2009a) that the intensified use of the deposit facility during the financial crisis indicates that banks in the euro area held excess reserves for precautionary motives. Instead, our argument is that as a consequence of large transaction costs, the ECB replaced the interbank market by assuming an intermediary function between surplus and deficit banks.[2] Transaction costs were large during the crisis because the crisis implied high bank asset losses combined with a high degree of uncertainty in how far individual banks were affected, and therefore also, about the soundness of potential transaction partners in the interbank market. As a consequence, it became

[1]The term "Eurosystem" stands for the institution which is responsible for monetary policy in the euro area, namely the ECB and the national central banks in the euro area. For the sake of simplicity, the terms "ECB" and "Eurosystem" are used interchangeably throughout this paper.

[2]This view is supported by recent evidence in European Central Bank (2009c). For the U.S. and the U.K., however, evidence indicates that precautionary motives for holding reserves do play a major role see Ashcraft, McAndrews, and Skeie (2009), Acharya and Merrouche (2009). For further empirical evidence on banks' bidding behaviour in the ECB's main refinancing operations during the financial crisis, see Eisenschmidt, Hirsch, and Linzert (2009).

much more difficult to find suitable counterparties in that market, so that transaction costs increased significantly.

The aim of the Eurosystem is to reverse its intermediary function and to reactivate interbank market transctions. According to our model, the obvious way to achieve this aim is to reduce interbank market transaction costs. However, this cannot be accomplished by the Eurosystem since it cannot reduce informational problems and uncertainties about the soundness of financial institutions. Our model shows that a central bank can reactivate interbank market transactions by making its transactions with the banking sector less attractive, which can be done, for example, by increasing requirements for collateral or by decreasing the rate on the deposit facility. However, as long as transactions in the interbank market are associated with high transaction costs, these activities will increase the banks' liquidity costs. In an economic and financial crisis, this increase usually conflicts with a central bank's aims from a monetary policy and financial stability perspective. Therefore, we suggest that the Eurosystem should make its transactions with the banking sector less attractive gradually over time, as informational problems and uncertainties become less and less significant.

The related literature on the liquidity management of credit institutions, central bank activities and the consequences for the interbank market can be divided into three groups. The first group focusses on the liquidity management of U.S. banks and the U.S. federal funds market before the financial crisis.[3] The second group concentrates on the banks' and central bank's behaviour and the interbank market in the euro area before the financial crisis.[4] Our paper belongs to the third group, which analyzes the credit institutions' and central banks' liquidity management during the financial crisis which started in August 2007. Allen, Carletti, and Gale (2009) discuss central bank measures to reduce the volatility of the interbank market rate. Eisenschmidt and Tapking (2009) analyze the evolution of liquidity risk premia in unsecured interbank markets. Bruche and Suarez (2010) and Heider, Hoerova, and Holthausen (2009) show that counterparty risk can lead to a decline in the transaction volume in the interbank market. While these contributions concentrate on particular aspects of the financial crisis, our paper aims at providing a simultaneous explanation for the strongly increased demand for central bank liquidity, the significant

[3]See, for example, Ho and Saunders (1985), Hamilton (1996), Clouse and Dow (1999, 2002), Furfine (2000), and Bartolini, Bertola, and Prati (2001, 2002).

[4]See, for example, Breitung and Nautz (2001), Ayuso and Repullo (2001, 2003), Ewerhart (2002), Bindseil (2002), Nautz and Oechssler (2003, 2006), Neyer and Wiemers (2004) and Neyer (2009).

use of the deposit facility, the decrease of the interbank market transaction volume, and the systematic decrease of the interbank market rate below the key policy rate in the euro area. Furthermore, respective policy implications are drawn. With this focus, our paper is complementary to Ashcraft, McAndrews, and Skeie (2009) who study similar aspects for the U.S. interbank market.

The rest of the paper is organized as follows. In section 2, we present the institutional background. Section 3 describes the stylized facts to be explained by our theoretical model which is presented in section 4. In section 5, we discuss the results and the policy implications. The last section summarizes the paper.

## 2 Institutional Background

Deposits that banks hold on their accounts with the central bank plus the currency they physically hold are the *reserves* of the banking sector. In the euro area, the needs for reserves arise from minimum reserve requirements and so-called autonomous liquidity factors, as banknotes in circulation.

The banking sector's needs for these reserves can only be satisfied by the Eurosystem. It has monopoly power over the creation of reserves. This allows the Eurosystem to steer the interest rate in the interbank market for reserves which is its operating target. For steering this interest rate, the Eurosystem assesses the needs for reserves and provides or absorbs the appropriate amount of liquidity. Important instruments for providing/absorbing reserves are the main refinancing operations (MROs), the longer-term refinancing operations, the fine-tuning operations and two standing facilities. The MROs are credit operations. They have a maturity of one week and are conducted weekly as either a fixed rate tender or a variable rate tender. For each MRO, the ECB calculates a benchmark allotment, which reflects the banking sector's liquidity needs during the maturity of the MRO if the reserve requirements are fulfilled smoothly over the reserve maintenance period.[5] In "normal" times, bids will be rationed if total bids exceed the benchmark allotment. A further source of reserves for the banking sector are *longer-term refinancing operations*. In "normal" times they are conducted once a month and have a maturity of

[5]The definition of the benchmark allotment reveals that although a single bank may fulfil its reserve requirements unevenly over the maintenance period (for fulfilling its reserve requirements, a credit institution can make use of averaging positions over the reserve maintenance period), the Eurosystem aims on aggregate a smooth fulfilment. The reason given is this enhances the buffer function of the minimum reserve system against transitory liquidity shocks (European Central Bank, 2002, p. 47).

three months. The *fine-tuning operations* are non-standardized instruments to provide or absorb liquidity. Concerning the two standing facilities one has to distinguish between a credit facility and a deposit facility. Both have an overnight maturity. On the initiative of the credit institutions, the credit facility provides liquidity, whereas the deposit facility absorbs liquidity. The interest rates on these facilities usually form a symmetric corridor around the MRO-rate. All credit operations with the Eurosystem have to be based on adequate collateral. Assets eligible as collateral must fulfil certain criteria defined by the Eurosystem.[6] During the financial crisis, which broke out in August 2007, times were no longer "normal" and the ECB adopted a couple of non-standard-measures comprising the following five building blocks.[7] (1) The Eurosystem fully satisfied the banks' demand for liquidity although it exceeded the benchmark allotment by far. (2) The list of assets eligible for use as collateral for credit operations with the Eurosystem was expanded. (3) The range of maturities of the longer-term refinancing operations was expanded up to one year. (4) The Eurosystem provided liquidity to the banking sector in the euro area in foreign currencies. (5) The Eurosystem started to purchase euro-denominated covered bonds.

In the interbank market for reserves, banks exchange deposits they hold on their accounts with the Eurosystem. This market thus reallocates the reserves originally provided by the central bank. One reason for this reallocation is that usually, the shortest frequency by which the Eurosystem provides reserves to the banking sector is one week, namely through its MROs. While the needs for reserves of the banking sector as a whole may not change significantly within one week, the needs for reserves of individual banks usually fluctuate daily. These fluctuations result from cash withdrawls and cash deposits by the banks' customers and from bank transfer payments. The reason for the latter is that the Eurosystem acts as a clearing institution by operating the most important interbank payment system in the euro area which implies that payments between banks are made by exchanging deposits on their reserve accounts at the Eurosystem. Another reason why banks exchange reserves on the interbank market is that not all banks borrow

[6]For a detailed description of the Eurosystem's monetary policy instruments including its minimum reserve system and for information on the collateral framework see European Central Bank (2008). For a detailed description of the Eurosystem's liquidity management see European Central Bank (2002).

[7]For a brief survey see, for example, Trichet (2009b). For a detailed description of the implementation of monetary policy by the Eurosystem in response to the financial crisis see European Central Bank (2009c).

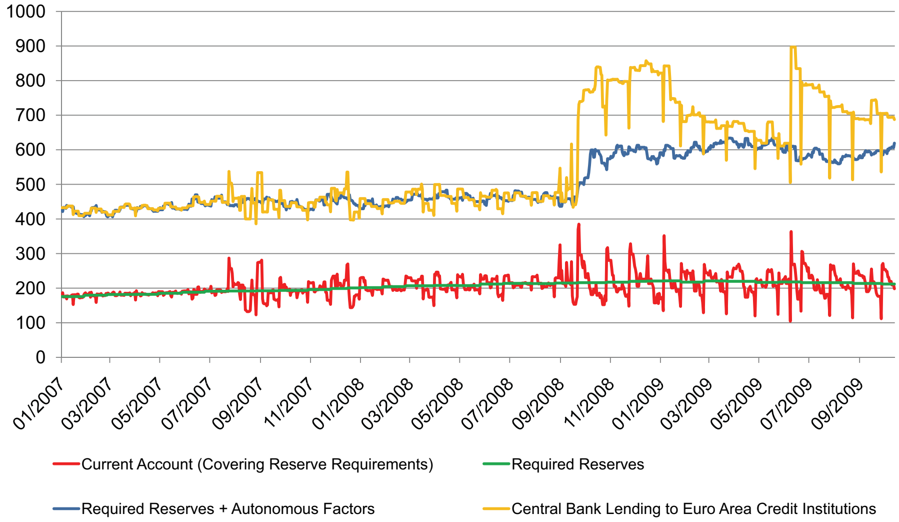


Figure 1: Needs of the Banking Sector for Reserves and Provision by the Eurosystem (EUR Billions). Data: ECB.

the reserves they need directly from the central bank but prefer to cover their needs for reserves exclusively in the interbank market.[8]

## 3 Stylized Facts

There are two phases of the financial crisis. The first phase began in August 2007, when the tensions in the euro area money market arose. The second phase started in September 2008, when the collapse of Lehman Brothers intensified the financial crisis.

Figure 1 shows that in the first phase of the financial crisis, the *actual* banking sector's needs for reserves resulting from reserve requirements and autonomous liquidity factors did not change significantly compared to the pre-crisis period. However, the Eurosystem changed the timing of its liquidity provisions.[9] Before the crisis, it covered the banking sector's actual liquidity needs so that the banking sector fulfilled its reserve requirements smoothly over a reserve maintenance period. The two lines representing the current account holdings and the reserve requirements almost coincide. However, from August 2007

[8]In the euro area, more than 1.700 banks are eligible to participate in the MROs. However, less than 500 banks actually take part in these operations (European Central Bank, 2007, p. 89). For a respective theoretical analysis see Neyer and Wiemers (2004).

[9]For a description of the implementation of monetary policy by the Eurosystem in response to the financial market tensions since August 2007 see European Central Bank (2009c).

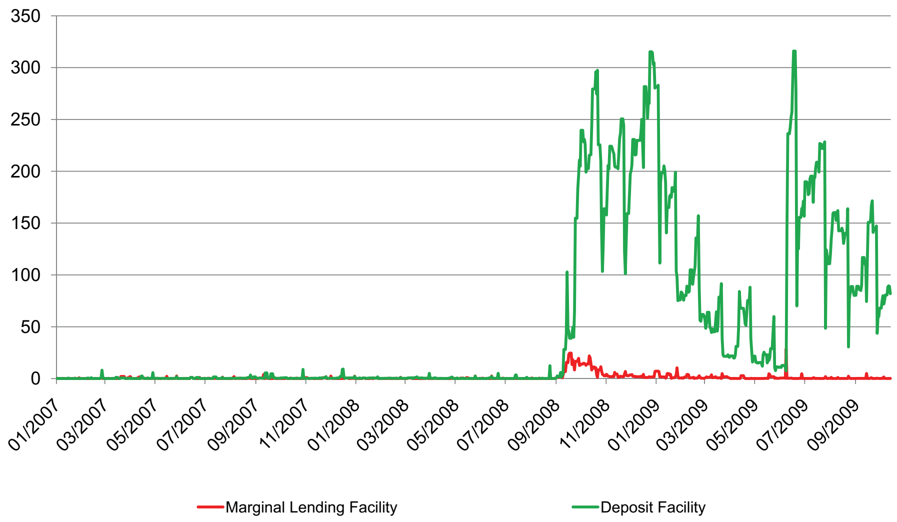


Figure 2: Needs of the Banking Sector for Reserves and Provision by the Eurosystem (EUR Billions). Data: ECB.

until September 2008, the Eurosystem allowed the credit institutions to "frontload" required reserves. At the beginning of a reserve maintenance period ample liquidity was provided, while over the course of the maintenance period the liquidity supply was gradually adjusted downwards. Over a maintenance period, the Eurosystem still only provided that amount of liquidity which corresponded to the banking sector's actual liquidity needs as Figure 1 reveals. This changed significantly with the beginning of the second phase of the financial crisis. The banking sector's demand for reserves strongly increased exceeding by far its actual liquidity needs, and the Eurosystem fully satisfied this increased demand for reserves by conducting fixed rate tenders with full allotment, which was one of the several non-standard measures, the Eurosystem adopted during the crisis. Consequently, outstanding central bank lending to the banking sector exceeded by far the banking sector's actual liquidity needs as shown in Figure 1.[10]

Figure 2 displays the use of the Eurosystem's standing facilities. Until the beginning of the second phase of the financial crisis, neither of the facilities was used intensively. However, with the beginning of the second phase banks started to place massive liquidity in

[10] Although also the liquidity needs of the banking sector increased with the beginning of the second phase, as shown in Figure 1, the increase in central bank lending to the banking sector was significantly higher. The increase in the banking sector's liquidity needs in the second phase is primarily the result of an increase in the autonomous liquidity factor "Liabilities to non-euro area residents denominated in euro" (see the Eurosystem's weekly financial statement, available at the ECB's website.)

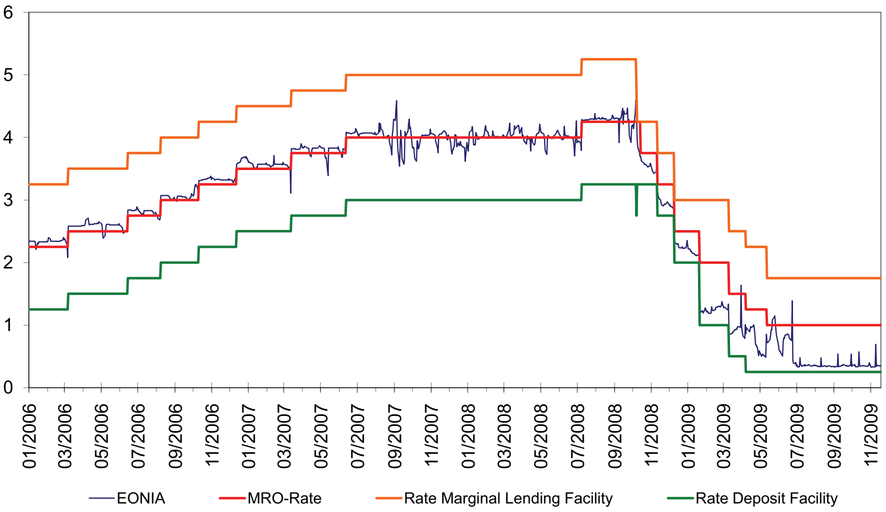


Figure 3: EONIA and Key ECB Interest Rates (Percentage). Data: ECB and Deutsche Bundesbank.

the deposit facility, while the claiming of the marginal lending facility remained rather low. At the same time, transaction in the interbank market for reserves decreased significantly (European Central Bank, 2009a,b). This decrease in interbank market activities came along with a systematic fall of the EONIA[11] below the MRO-rate (see Figure 3). Note that usually, there is a positive spread between the interbank market rate and MRO-rate.[12]

To sum up, the second phase of the financial crisis, which started in September 2008, was associated with (1) a strong increase in the banking sector's demand for reserves in the Eurosystem's tender procedures, (2) a strong increase in the use of the Eurosystem's deposit facility, whereas the use of the marginal lending facility did not increase significantly, (3) a strong decrease in transactions in the interbank market for reserves, and (4) a systematic fall of the EONIA below the MRO-rate.

[11]EONIA is the abbreviation for *E*uro *O*vernight *I*ndex *A*verage. It is a market index computed as the weighted average of overnight unsecured lending transactions undertaken by a representative panel of banks. For more information on this reference rate see www.euribor.org.

[12]For respective empirical analyses see, for example, Nyborg, Bindseil, and Strebulaev (2002), Ayuso and Repullo (2003), Ejerskov, Moss, and Stracca (2003), Nyborg, Bindseil, and Strebulaev (2002), and Neyer and Wiemers (2004). For a theoretical explanation see, for example, Ayuso and Repullo (2003) and Neyer and Wiemers (2004).

# 4 The Model

## 4.1 Framework

In this section, we introduce a theoretical model which can explain the just described stylized facts. It replicates the main institutional features of the euro area market for reserves. There is a central bank and a large number of risk-neutral, price-taking commercial banks. Each commercial bank faces autonomous liquidity needs $A$. A bank with $A > 0$ has a liquidity deficit while a bank with $A < 0$ has a liquidity surplus. Each single bank can obtain and place liquidity at the central bank. Furthermore, each bank may borrow and lend liquidity in the interbank market

There are two ways of obtaining liquidity from the central bank. Firstly, a commercial bank can take part in an MRO and borrow the amount $K \geq 0$ at the rate $i^{MRO}$. Secondly, it can use a standing credit facility offered by the central bank and borrow the amount $CF \geq 0$ at the rate $i^{CF}$. Both credit operations have to be based on adequate collateral. The costs of holding collateral are equal to $\alpha > 0$ per unit of liquidity. Consequently, borrowing $K$ from the central bank in the MRO costs

$$C^{MRO} = \left(i^{MRO} + \alpha\right) K \tag{1}$$

and borrowing the amount $CF$ in the credit facility costs

$$C^{CF} = \left(i^{CF} + \alpha\right) CF. \tag{2}$$

The central bank also offers a deposit facility. A commercial bank can place liquidity $DF \geq 0$ in this facility at the rate $i^{DF}$ so that costs of the deposit facility are

$$C^{DF} = -i^{DF} DF. \tag{3}$$

We assume that the rates on the facilities form a symmetric corridor around the MRO-rate, so that $i^{MRO} = \frac{i^{CF}+i^{DF}}{2}$, with $i^{DF} < i^{MRO} < i^{CF}$.

Each commercial bank can borrow and lend liquidity in the interbank market. A bank's position in that market is $B$. If $B > 0$, the bank will borrow liquidity at the rate $i^{IBM}$. If $B < 0$, the bank will lend at this rate. Trading in the interbank market involves transaction costs which are equal to $\frac{1}{2}\gamma B^2$ with $\gamma > 0$. This quadratic form is a common approach of modeling transaction costs in the interbank market (see, for example, Campbell, 1987; Bartolini, Bertola, and Prati, 2001). It reflects increasing marginal costs of searching for banks with matching liquidity needs and those marginal costs resulting from the need to split large transactions into many small ones to work around credit lines. Therefore, a bank's costs in the interbank market are

$$C^{IBM} = i^{IBM} B + \frac{1}{2}\gamma B^2. \tag{4}$$

There are two types $j = a, b$ of commercial banks which differ with respect to their autonomous liquidity needs. Type $a$ faces an autonomous liquidity deficit ($A^a > 0$) whereas type $b$ has an autonomous liquidity surplus ($A^b < 0$). Half of the population is of type $a$ while the rest is of type $b$. In the following, we will refer to these two types simply as bank $a$ and bank $b$. The extent of either bank's surplus or deficit is uncertain and depends on the state of the world. There are two states of the world, each occurring with probability $\frac{1}{2}$. Bank $a$ has a relatively small deficit $A^a_L$ in state 1 and a relatively large deficit $A^a_H > A^a_L$ in state 2. If bank $a$ faces a large deficit, bank $b$ faces a large surplus and vice versa. In each state of the world, bank $a$'s deficit is higher than bank $b$'s surplus, so that there is always a liquidity deficit at the aggregate level which can only be covered by the central bank. We assume that this aggregate deficit $D$ is certain, it is thus the same in either state of the world. The numerical example given in Table 1 exemplifies the interrelation between the banks' autonomous liquidity needs which is formally given by $A^a_L + A^b_H = A^a_H + A^b_L =: D$.

| | Bank $a$ | Bank $b$ | Total |
|---|---|---|---|
| State 1 | $A^a_L = 80$ | $A^b_H = -60$ | $D = 20$ |
| State 2 | $A^a_H = 100$ | $A^b_L = -80$ | $D = 20$ |

Table 1: Autonomous Liquidity Needs in the Different States of the World

The sequence of moves is as follows: First, each bank chooses and receives the amount of liquidity $K$ that it wishes to obtain in the central bank's MRO. We thus assume that the central bank fully satisfies each bid $K$ of the banks. Then, the state of the world realizes, each bank type learns its autonomous liquidity needs $A$, and chooses the amount

$B$ that it wishes to borrow or lend in the interbank market.[13] Simultaneously, it decides whether or not to use the credit facility or the deposit facility offered by the central bank.

A bank will use the credit facility if its transaction in the interbank market implies that it is left with a liquidity shortage, i.e. if $A - K - B > 0$. Then, the bank will cover this deficit by borrowing from the central bank via the credit facility ($CF = A - K - B$). A bank will use the deposit facility if its transaction in the interbank market implies that it is left with excess liquidity, i.e. if $A - K - B < 0$. Then, it will place this surplus amount in the deposit facility ($DF = B + K - A$). Using the equations (2) and (3), we then obtain for the costs of using the facilities

$$C^F = \begin{cases} \left(i^{CF} + \alpha\right)(A - K - B) & \text{if} \quad A - K - B > 0 \\ 0 & \text{if} \quad A - K - B = 0 \\ -i^{DF}(B + K - A) & \text{if} \quad A - K - B < 0 \end{cases} . \tag{5}$$

Each bank aims at minimizing its total liquidity costs by choosing its optimal net borrowing from the central bank and its optimal transactions in the interbank market. We solve this optimization problem backwards. First, we determine a bank's optimal behavior in the interbank market and thus its optimal use of the facilities offered by the central bank (second stage). Then, we derive the banks' optimal borrowing in the central bank's MRO (first stage).

### 4.2 Optimal Transactions in the Interbank Market (Second Stage)

This section concentrates on the second stage of the model. We proceed in two steps. First, we determine a bank's optimal transaction in the interbank market taking the interest rate $i^{IBM}$ as given. Then, we derive the equilibrium interest rate $i^{IBM^*}$.

After the state of the world has been realized, a bank knows its actual liquidity needs $A$ and has to decide whether and how much to trade in the interbank market, and thus, in how far to use the facilities offered by the central bank. As the bank aims at minimizing its liquidity costs, its optimization problem is

$$C^{IBM} + C^F =: f(B) \to \min . \tag{6}$$

[13] Note that whenever it is not necessary to distinguish between $A_H$ and $A_L$, we drop the subscript from our notation.

Note that we can restrict attention to $i^{IBM} \in \left[i^{DF}; i^{CF} + \alpha\right]$. If the interbank rate were smaller than $i^{DF}$, no bank would be willing to supply credit as it would pay more to place excess liquidity in the deposit facility. If it were larger than $i^{CF} + \alpha$, no bank would demand credit in the interbank market. Instead, banks would use the credit facility to overcome a liquidity shortage. Solving the bank's optimization problem (6) and denoting the optimal transaction in the interbank market by $B^{opt}$, we obtain

**Lemma 1:** *In the interbank market, a bank*

- *with a liquidity gap $A - K > 0$ will choose:*

$$B^{opt} := \begin{cases} A - K & \text{if } \ i^{IBM} + \gamma(A - K) \leq i^{CF} + \alpha \\ \frac{i^{CF}+\alpha-i^{IBM}}{\gamma} < A - K & \text{if } \ i^{IBM} + \gamma(A - K) > i^{CF} + \alpha \end{cases}, \tag{7}$$

- *with excess liquidity $K - A \geq 0$ will choose:*

$$B^{opt} := \begin{cases} -\frac{i^{IBM}-i^{DF}}{\gamma} > -(K - A) & \text{if } \ i^{IBM} - \gamma(K - A) < i^{DF} \\ -(K - A) & \text{if } \ i^{IBM} - \gamma(K - A) \geq i^{DF} \end{cases}. \tag{8}$$

**Proof:** see appendix.

To interpret the Lemma, consider a bank which faces a liquidity gap $A - K > 0$ after the state of the world has been realized. The bank can close the gap by borrowing $A - K$ in the interbank market. The marginal cost of this transaction, consisting of the interest payment and the transaction costs, is equal to $i^{IBM} + \gamma\,(A - K)$. As long as this marginal cost is (weakly) smaller than the marginal cost $i^{CF} + \alpha$ of the credit facility, it does not pay to use the credit facility. Accordingly, the bank will borrow $B^{opt} = A - K$ in the interbank market and will not use the credit facility. This, however, is no longer true if the marginal cost of closing the total liquidity gap through the interbank market exceeds the marginal cost $i^{CF} + \alpha$ of using the credit facility. Then, the bank will use the interbank market and/or the credit facility, i.e. $0 \leq B^{opt} < A - K$. The bank will cut borrowing in the interbank market until the marginal cost of borrowing in the interbank market equals the marginal cost of the credit facility ($i^{IBM} + \gamma B^{opt} = i^{CF} + \alpha$).

The case of a bank with excess liquidity $K - A \geq 0$ can be interpreted along the same lines. Suppose that this bank lends its excess liquidity in the interbank market. Then, the marginal net return is equal to $i^{IBM} - \gamma\,(K - A)$. If this marginal net return is (weakly) higher than the marginal return $i^{DF}$ of the deposit facility, the bank has no reason to use the deposit facility, and chooses $B^{opt} = -(K - A)$. Otherwise, if $i^{IBM} - \gamma(K - A)$ falls short of $i^{DF}$, the bank will cut lending in the interbank market until the marginal net return of lending equals $i^{DF}$.

Lemma 1 has implications for the transaction volume $|B|$ in the interbank market. On the one hand, a bank is willing to borrow some amount equal to $|B|$ in this market only if the marginal cost $i^{IBM} + \gamma|B|$ of borrowing does not exceed the marginal cost $i^{CF} + \alpha$ of the credit facility. On the other hand, a bank is willing to lend an amount equal to $|B|$ only if the marginal net return $i^{IBM} - \gamma|B|$ is at least as large as the marginal return $i^{DF}$ of the deposit facility. Both conditions imply that the transaction volume in the interbank market cannot exceed a specific amount

$$|B| \leq \frac{\frac{1}{2}(i^{CF} + \alpha - i^{DF})}{\gamma} =: |B|^{marg}, \tag{9}$$

where $|B|^{marg}$ denotes the transaction volume, for which the marginality condition of the borrower as well as the marginality condition of the lender is met with equality.

Recall that in either state of the world, bank $b$ faces an autonomous liquidity surplus ($A^b < 0$). Consequently, also after a possible bidding in the MRO, bank $b$ has excess liquidity $K^b - A^b$. This, together with Lemma 1, implies that, if at all, bank $b$ will *supply* liquidity in the interbank market ($B^{b^{opt}} \leq 0$). Therefore, the market clearing condition $B^{a^{opt}} + B^{b^{opt}} = 0$ requires $B^{a^{opt}} \geq 0$, and Lemma 1 leads us to

**Lemma 2:** *The equilibrium on the interbank market has the following properties:*

(a) *If $A^a - K^a \leq 0$, then:*

$$i^{IBM^*} = i^{DF} \quad \textit{and} \tag{10}$$

$$|B|^* = 0. \tag{11}$$

(b) *If $A^a - K^a \in ]0, K^b - A^b[$, then:*

$$i^{IBM^*} = i^{DF} + \gamma|B|^* \quad \textit{and} \tag{12}$$

$$|B|^* = \min\left\{A^a - K^a, |B|^{marg}\right\}. \tag{13}$$

(c) *If* $A^a - K^a > K^b - A^b$ *then:*

$$i^{IBM*} = i^{CF} + \alpha - \gamma|B|^* \quad and \tag{14}$$
$$|B|^* = \min\left\{K^b - A^b, |B|^{marg}\right\}. \tag{15}$$

(d) *If* $A^a - K^a = K^b - A^b$ *then:*

$$i^{IBM*} \in \left[i^{DF} + \gamma|B|^*, i^{CF} + \alpha - \gamma|B|^*\right] \quad and \tag{16}$$
$$|B|^* = \min\left\{K^b - A^b, |B|^{marg}\right\}. \tag{17}$$

**Proof:** see appendix.

Bank $b$ will not demand for credit in the interbank market. Based on this, Lemma 2 reveals that depending on the liquidity position of bank $a$, we can distinguish four scenarios, (a) to (d), when discussing the interbank market equilibrium.

In scenario (a), bank $a$ has excess liquidity so that it does not demand credit in the interbank market either. Accordingly, there will be no transaction in that market, $|B|^* = 0$. The equilibrium interbank rate is equal to the marginal return $i^{DF}$ of the deposit facility in this scenario. This ensures that neither bank $a$ nor bank $b$ supplies or demands credit. Instead, they deposit their excess liquidity in the deposit facility.

In scenario (b), bank $a$ faces a liquidity gap $A^a - K^a$ which is smaller than the liquidity surplus $K^b - A^b$ of bank $b$. Consequently, there is an aggregate liquidity surplus. In this case, as long as the liquidity gap of bank $a$ does not exceed $|B|^{marg}$, bank $a$ will close the gap exclusively by borrowing in the interbank market. Otherwise, it will borrow $|B|^{marg}$ in the interbank market and use the credit facility in order to close the remaining liquidity gap. Due to the aggregate liquidity surplus, lenders compete for borrowers. As a consequence, the interbank market rate will be bid down until the marginal net return $i^{IBM} - \gamma|B|$ of lending in the interbank market is equal to the marginal return $i^{DF}$ of the deposit facility. We thus have $i^{IBM*} = i^{DF} + \gamma|B|^*$, and bank $b$ will deposit some of its excess liquidity in the deposit facility.

Scenario (c) is characterized by an aggregate liquidity deficit as the liquidity gap $A^a - K^a$ of bank $a$ is larger than the liquidity surplus $K^b - A^b$ of bank $b$. If this liquidity surplus does not exceed the transaction volume $|B|^{marg}$, bank $b$ will lend all of its excess liquidity to bank $a$, and bank $a$ will use the credit facility to satisfy its remaining liquidity needs.

If, however, $K^b - A^b$ exceeds $|B|^{marg}$, bank $b$ will lend $|B|^{marg}$ to bank $a$. Then, the credit facility as well as the deposit facility will be used. The former will be used by bank $a$, the latter by bank $b$. In this scenario, borrowers compete for obtaining liquidity. Therefore, the interbank rate will be bid up until the marginal costs of borrowing in the interbank market $i^{IBM} + \gamma|B|$ equals the marginal cost $i^{CF} + \alpha$ of using the credit facility.

In the last scenario (d), there is neither an aggregate liquidity deficit nor a surplus. Like in the scenario before, bank $b$ will lend either its complete excess liquidity $K^b - A^b$ or an amount equal to $|B|^{marg}$ to bank $a$, depending on which amount is smaller. In this scenario, no bank will have the "power" to bid up or down the interbank market rate to the marginal costs of its counterparty, and we can only say that $i^{IBM}$ will be somewhere in the interval between $i^{DF} + \gamma|B|^*$ and $i^{CF} + \alpha - \gamma|B|^*$.

### 4.3 Optimal Borrowing in the MRO (First Stage) and Equilibrium

This section concentrates on the first stage of the model. At this stage, the banks choose the amount of liquidity $K \geq 0$ they bid for in the MRO. They do not yet know which state of the world will occur later on. That is, when deciding upon $K$, the banks are uncertain whether their autonomous liquidity needs $A$ will be small ($A = A_L$) or large ($A = A_H$).

The amount of liquidity $K$ a bank borrows in the MRO also determines the amount of liquidity it borrows or lends in the interbank market and its usage of the facilities. Consequently, the decision problem of a bank reads:

$$\begin{aligned} C^{MRO} \quad &+ \quad \frac{1}{2}\left[C_L^{IBM}\left(B_L^{opt}\right) + C_L^F\left(B_L^{opt}\right)\right] \\ &+ \quad \frac{1}{2}\left[C_H^{IBM}\left(B_H^{opt}\right) + C_H^F\left(B_H^{opt}\right)\right] =: g(K) \rightarrow \min_{K, K\geq 0} \end{aligned} \tag{18}$$

where $B_L^{opt}$ ($B_H^{opt}$) denotes the bank's optimal transactions in the interbank market in case of low (high) liquidity needs. Solving this optimization problem and considering that in equilibrium, each bank's bid in the MRO must be optimal given that the implied interest rate in the interbank market in either state is consistent with these bids, we obtain

**Proposition:** *Define*

$$\bar{\gamma} := \frac{\frac{1}{2}(i^{MRO} + \alpha - i^{DF})}{E[A^a] - D}, \tag{19}$$

$$\bar{\bar{\gamma}} := \frac{\frac{1}{2}(i^{CF} - i^{DF})}{A_H^a - A_L^a}, \quad \textit{and} \tag{20}$$

$$\widehat{\gamma} := \frac{\frac{1}{2}(i^{CF} + \alpha - i^{DF})}{A_H^a - D}. \tag{21}$$

*Suppose that* $\bar{\gamma} < \widehat{\gamma}$*, then, the overall equilibrium has the following properties:*

(a) *If* $\gamma \leq \bar{\gamma}$*:*

$$\begin{aligned}
K^{a^*} &= D, \\
K^{b^*} &= 0, \\
|B_1|^* &= A_L^a - D, \\
|B_2|^* &= A_H^a - D, \\
DF_1^* = DF_2^* &= 0, \\
CF_1^* = CF_2^* &= 0, \\
i^{IBM^*} &\in [i^{DF} + \gamma|B|^*, i^{CF} + \alpha - \gamma|B|^*], \\
E[i^{IBM^*}] &= i^{MRO} + \alpha - \gamma(E[A^a] - D).
\end{aligned}$$

(b) *If* $\bar{\gamma} < \gamma < \bar{\bar{\gamma}}$*:*

$$\begin{aligned}
K^{a^*} &= E[A^a] - \frac{\frac{1}{2}(i^{MRO}+\alpha-i^{DF})}{\gamma}, \\
K^{b^*} &= 0, \\
|B_1|^* &= \frac{\frac{1}{2}(i^{MRO}+\alpha-i^{DF})}{\gamma} - \tfrac{1}{2}(A_H^a - A_L^a), \\
|B_2|^* &= \frac{\frac{1}{2}(i^{MRO}+\alpha-i^{DF})}{\gamma} + \tfrac{1}{2}(A_H^a - A_L^a), \\
DF_1^* = DF_2^* &= E[A^a] - D - \frac{\frac{1}{2}(i^{MRO}+\alpha-i^{DF})}{\gamma}, \\
CF_1^* = CF_2^* &= 0, \\
i_1^{IBM^*} &= \tfrac{1}{2}i^{DF} + \tfrac{1}{2}(i^{MRO} + \alpha) - \tfrac{1}{2}\gamma(A_H^a - A_L^a), \\
i_2^{IBM^*} &= \tfrac{1}{2}i^{DF} + \tfrac{1}{2}(i^{MRO} + \alpha) + \tfrac{1}{2}\gamma(A_H^a - A_L^a).
\end{aligned}$$

(c) *If* $\gamma \geq \bar{\bar{\gamma}}$*:*

$$\begin{aligned}
K^{a^*} &= A_L^a - \frac{\frac{1}{2}\alpha}{\gamma}, \\
K^{b^*} &= 0, \\
|B_1|^* &= \frac{\frac{1}{2}\alpha}{\gamma}, \\
|B_2|^* &= \frac{\frac{1}{2}(i^{CF}+\alpha-i^{DF})}{\gamma}, \\
DF_1^* &= A_L^a - D - \frac{\frac{1}{2}\alpha}{\gamma}, \\
DF_2^* &= A_H^a - D - \frac{\frac{1}{2}(i^{CF}+\alpha-i^{DF})}{\gamma}, \\
CF_1^* &= 0, \\
CF_2^* &= A_H^a - A_L^a - \frac{\frac{1}{2}(i^{CF}-i^{DF})}{\gamma}, \\
i_1^{IBM^*} &= i^{DF} + \tfrac{1}{2}\alpha, i_2^{IBM^*} = i^{MRO} + \tfrac{1}{2}\alpha.
\end{aligned}$$

**Proof:** see appendix.

The Proposition reveals that depending on the transaction cost parameter $\gamma$ three regimes can be distinguished. In regime (a) transaction costs are small ($\gamma \leq \bar{\gamma}$), in regime (b) transaction costs are large ($\bar{\gamma} < \gamma < \bar{\bar{\gamma}}$), and in regime (c) they are extremely large ($\gamma \geq \bar{\bar{\gamma}}$). In what follows, we will first use Figure 4 to provide an overview of the results given in the Proposition before discussing them in more detail. Panel (i) shows that the surplus bank $b$ does not participate in the MRO in either regime. However, bank $a$'s borrowing in the MRO differs across the three regimes. The maximum possible amount bank $a$ can borrow in the interbank market is bank $b$'s surplus. In regime (a) transaction costs in the interbank market are that low that bank $a$ borrows this maximum amount (panel (iii)) and covers the remaining deficit, which corresponds to the aggregate deficit $D$, by borrowing in the MRO (panel i). Consequently, in regime (a) none of the facilities is used (panel (ii) and (iv)). In regime (b) large transaction costs on the interbank market imply that bank $a$ prefers to borrow more in the MRO and less in the interbank market so that bank $b$ is no longer able to place its total liquidity surplus in the interbank market and thus uses the deposit facility. Increasing transaction costs induce bank $a$ to cover more and more of its liquidity deficit in the MRO so that bank $b$ places more and more liquidity in the deposit facility. In regime (c) extremely large transaction costs imply that

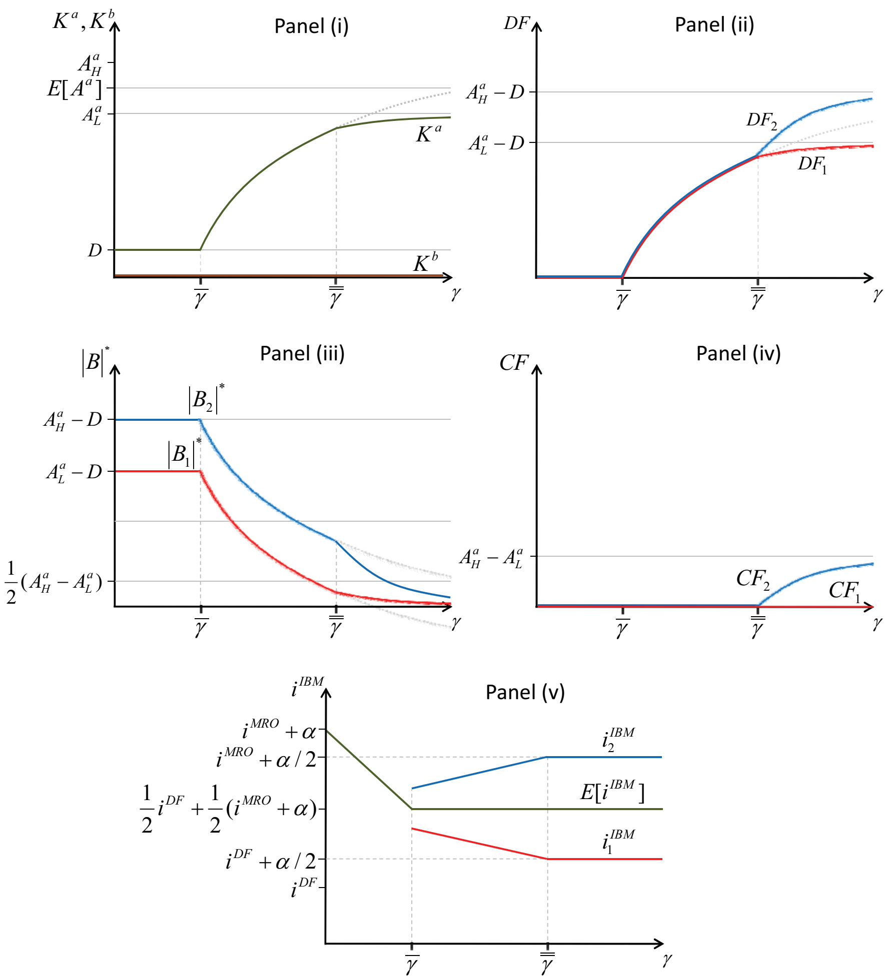


Figure 4: Borrowing in the MRO, Transactions in the Interbank Market, Use of the Facilities, and the Interbank Market Rate against Transactions Costs

bank $a$ will even prefer to use the credit facility instead of borrowing in the interbank market if the second state of the world is realized. Since in the regimes (b) and (c) more than the aggregate liquidity deficit $D$ is borrowed in the MRO, there is excess liquidity in the interbank market which brings down the interbank market rate below the MRO-rate.

Let us now discuss the three regimes in some more detail. Bank $b$ does not participate in the MRO in either regime. It never pays for the surplus bank $b$ to borrow from the central bank in order to place the additional liquidity in the interbank market or in the deposit facility since for any $K^b \geq 0$ marginal costs are higher than expected (net) marginal benefits. In regime (a), in which bank $a$ borrows bank $b$'s total surplus in the interbank market and the aggregate deficit $D$ in the MRO, there is neither an aggregate liquidity deficit nor an aggregate liquidity surplus after bank $a$'s bidding in the MRO. Therefore, there is no market power on either side of the market so that we can only say that the interbank market rate will lay between the net marginal opportunity revenues of lending in the interbank market $i^{DF} + \gamma|B|^*$ and the marginal opportunity costs of borrowing in the interbank market $i^{CF} + \alpha - \gamma|B|^*$. However, from an ex ante perspective, i.e. *before* the state of the world has been realized, the expected interbank rate $E[i^{IBM}]$ is determined: To ensure that bank $a$ borrows $D$ in the MRO, $E[i^{IBM}]$ must adjust until the marginal cost of the MRO equals the expected marginal cost of borrowing bank $b$'s surplus in the interbank market ($i^{MRO} + \alpha = E[i^{IBM}] + \gamma(E[A^a] - D)$). At the same time, the expected interbank rate must be large enough to induce bank $b$ to lend its surplus instead of using the deposit facility. That is, the expected marginal net return of lending must be (weakly) higher than the marginal return of the deposit facility ($E[i^{IBM}] - \gamma(E[A^a] - D) \geq i^{DF}$). These two requirements for $E[i^{IBM}]$ result in the condition $\gamma \leq \bar{\gamma}$ for transacting bank $b$'s total surplus in the interbank market.[14] Since in regime (a), this condition is met, bank $a$ bids for the aggregate deficit in the MRO and *expects* to cover its total remaining deficit, which corresponds to bank $b$'s total surplus, in the interbank market. However, the banks do not only *expect* to trade bank $b$'s total surplus in the interbank market from an ex ante perspective. From an ex post perspective, i.e. *after* the state of the world has occurred, they actually *do* exchange the complete liquidity surplus of bank $b$. This is particularly crucial in state 2. In this state, bank $b$'s actual surplus, and therefore, actual transaction costs in the interbank market, are rather large so that it might be favorable for bank $a$ to use the credit facility instead of covering its total remaining deficit in the interbank market. Bank $a$ refrains from doing so and borrows $A^a_H - D$ from bank $b$ via the interbank market in state 2 only if transaction costs are sufficiently small with $\gamma \leq \widehat{\gamma}$, where the threshold $\widehat{\gamma}$ is defined by (21).[15] This condition is met in regime (a) since we restrict our analysis to the case $\bar{\gamma} < \widehat{\gamma}$. We will comment on this restriction at the end of this section. Note that in this regime, an increase in $\gamma$ leads to a decrease in $E[i^{IBM}]$. Intuitively,

[14] To see this, rearrange the first requirement to $E[i^{IBM}] = i^{MRO} + \alpha - \gamma(E[A^a] - D)$ and the second requirement to $E[i^{IBM}] \geq i^{DF} + \gamma(E[A^a] - D)$ so that we must have $i^{MRO} + \alpha - \gamma(E[A^a] - D) \geq i^{DF} + \gamma(E[A^a] - D)$ or $\gamma \leq \frac{\frac{1}{2}(i^{MRO} + \alpha - i^{DF})}{E[A^a] - D} =: \bar{\gamma}$

[15] To see this, recall from (9) that the transaction volume in the interbank market must satisfy $|B| \leq \frac{\frac{1}{2}(i^{CF} + \alpha - i^{DF})}{\gamma}$ so that $|B| = A^a_H - D$ is feasible only if $\gamma \leq \frac{\frac{1}{2}(i^{CF} + \alpha - i^{DF})}{A^a_H - D} =: \widehat{\gamma}$.

higher transaction costs make the interbank market less attractive for bank $a$. Therefore, the expected interbank rate must decrease to offset the higher transaction costs.

Regime (b) with $\bar{\gamma} < \gamma < \bar{\bar{\gamma}}$ is characterized by relatively large transaction costs so that at the point $K^a = D$, marginal costs of borrowing in the MRO are lower than those of using the interbank market. Therefore, it is too expensive for the banks to exchange bank $b$'s total liquidity surplus. As a consequence, bank $a$ expands borrowing in the MRO beyond the aggregate deficit $D$ and bank $b$ has to use the deposit facility. However, the credit facility is not used. To understand this equilibrium, note that due to the aggregate liquidity surplus, the interbank rate will always bid down until bank $b$'s marginal net revenue of lending is equal to the marginal revenue of the deposit facility ($i^{IBM} - \gamma(A^a - K^a) = i^{DF}$), so that from an ex ante perspective, we have $E[i^{IBM}] = i^{DF} + \gamma(E[A^a] - K^a)$. Moreover, bank $a$ borrows in the MRO until the marginal costs equal the expected marginal cost of borrowing in the interbank market ($i^{MRO} + \alpha = E[i^{IBM}] + \gamma(E[A^a] - K^a)$). Putting these two conditions together results in the optimal $K^a$. The implied transaction volumes $|B_1|^*$ and $|B_2|^*$ do not exceed $|B|^{marg}$. Therefore, from an ex post perspective, bank $a$ has no reason to use the credit facility. In regime (b), an increase in $\gamma$ makes the interbank market less attractive. Therefore, bank $a$'s borrowing in the MRO increases and the respective actual transaction volumes in the interbank market decrease in $\gamma$. Consequently, bank $b$ places more and more liquidity in the deposit facility. Concerning the interbank market rate, note that a one percent increase in $\gamma$ lowers the expected transaction volume $E[|B|^*]$ in the interbank market by one percent so that the marginal expected transaction costs $\gamma E[|B|^*]$ are independent of $\gamma$. Accordingly, the expected interbank rate, which equals the sum of the marginal return of the deposit facility and the marginal expected transaction costs ($E[i^{IBM}] = i^{DF} + \gamma E[|B|^*]$), does not depend on $\gamma$ either. This, however, is not true for the respective interbank market rates in the two states as Figure 4 illustrates. When $\gamma$ increases by one percent, $|B|$ declines by the same absolute amount in both states. Therefore, since $|B_1|^* < |B_2|^*$, a one percent increase in $\gamma$ decreases $|B_1|^*$ ($|B_2|^*$) by more (less) than one percent so that the interbank rate falls (raises) in state 1 (2).

In the last regime (c), in which transaction costs are extremely large with $\gamma \geq \bar{\bar{\gamma}}$, bank $a$ will again change its behavior. Crucial is that in this regime transaction costs are even that high that in the second state of the world, in which bank $a$ has high liquidity needs, it prefers to cover at least parts of its remaining deficit by using the credit facility. The credit facility will only be used in state 2 since the liquidity deficit that bank $a$ faces in state 1 is certain. Independently of whether state 1 or 2 occurs, bank $a$ has at least liquidity needs equal to $A_L^a$ (see Table 1), and it is obviously less costly to cover certain liquidity needs by borrowing in the MRO than by using the credit facility. Therefore, in regime (c), bank $a$ borrows in the MRO until the marginal costs $i^{MRO} + \alpha$ of the MRO satisfy:

$$i^{MRO} + \alpha = \frac{1}{2}(i_1^{IBM} + \gamma(A_L^a - K^a)) + \frac{1}{2}(i^{CF} + \alpha), \tag{22}$$

where the first term on the RHS reflects the marginal costs in the interbank market in state 1 and the second term reflects the marginal costs of the credit facility in state 2. Like in regime (b), there will be an aggregate liquidity surplus so that we have $i_1^{IBM} = i^{DF} + \gamma(A_L^a - K^a)$. Together with (22), this leads to the optimal $K^a$. Relative to a scenario without a credit facility (see the respective dashed line in Figure 4), the bank will thus bid less in the MRO. This is because the existence of the credit facility makes the MRO relatively less attractive. Due to the smaller amount obtained in the MRO, bank $a$ will borrow more from bank $b$ in state 1, so that there is less usage of the deposit facility in this state compared to a situation without credit facility. In state 2, however, bank $b$ puts even more liquidity in the deposit facility. In regime (c), the interest rate in the interbank market does not change in $\gamma$ because a one percent increase in $\gamma$ decreases the transaction volume $|B|$ in the interbank market by one percent in both states of the world. Therefore, the marginal transaction costs $\gamma|B|$ and thus the interest rate remain unchanged.

Let us conclude this section with a brief comment on the restriction $\bar{\gamma} < \widehat{\gamma}$, that we introduced in the Proposition. If this condition were not met, the results of regime (b) would change. Bank $a$ would borrow the aggregate deficit $D$ in the MRO in regime (b) and it would rely on the credit facility to some extent in the second state of the world. That is, relative to the scenario described in the Proposition, there would be less bidding in the MRO and a stronger tendency to use the credit facility. This, however, does not fit to the stylized facts observed in the euro area. Therefore, we abstain here from discussing the case $\bar{\gamma} \geq \widehat{\gamma}$ in detail.[16]

[16] We do, however, derive the equilibrium for this case formally in the proof of the proposition.

# 5 Discussion

## 5.1 Explanation of the Stylized Facts

In section 3 we identified the following four stylized facts for the second phase of the financial crisis, i.e. after the collapse of Lehman Brothers. (1) A strong increase in the banking sector's demand for reserves in the Eurosystem's tender procedures, (2) a strong increase in the use of the Eurosystem's deposit facility, whereas the use of the marginal lending facility did not increase significantly, (3) a strong decrease in transactions on the interbank market for reserves, and (4) a systematic fall of the EONIA below the MRO-rate. These stylized facts correspond exactly with our model results described by regime (b). Therefore, we argue that the financial crisis, especially after the collapse of Lehman Brothers, implied a strong increase in transaction costs in the interbank market for reserves. The reason is that the financial crisis led to severe bank-asset losses combined with a high degree of uncertainty in how far and to what extent individual banks were affected by these losses. This implied that it became more difficult to find suitable counterparties on the interbank market. The increased transaction costs in the interbank market implied that for deficit banks it became more attractive to cover their liquidity needs by participating in the Eurosystem's fixed rate tender procedures than by borrowing in the interbank market. As a consequence, the demand for reserves in the tender procedures increased significantly. Since this demand was totally satisfied by the ECB, transactions in the interbank market fell significantly and the amount of outstanding central bank credits to the banking sector exceeded by far the banking sector's liquidity needs resulting from the minimum reserve requirements and the autonomous factors. Consequently, at the aggregate level, there was excess liquidity in the banking sector so that the surplus banks had to place liquidity in the Eurosystem's deposit facility and the EONIA fell below the fixed MRO-rate near to its lower bound which is the rate on the deposit facility.

There are two aspects which are particularly noteworthy. Firstly, according to our model, the strong use of the deposit facility in the second phase of the financial crisis is not the result of *precautionary motives.* If banks held central bank balances because of precautionary motives, they would cover *uncertain* liquidity needs by borrowing in the Eurosystem's tender procedures. However, as long as the probability of high liquidity needs is not sufficiently higher than the probability of low liquidity needs (in our model these probabilities are assumed to be the same), this behavior is not rational. Borrowing in the tender procedures and hording the liquidity in the deposit facility as a precaution is more expensive than using the credit facility if necessary. According to our model, the strong usage of the deposit facility is due to the fact that for deficit banks it is more attractive to borrow from the central bank than in the interbank market which implies that surplus banks are not able to place their excess liquidity at adequate conditions in the interbank market. For them too, transacting with the central bank is the more attractive alternative. This leads us to the second aspect. Since surplus banks place excess liquidity at the Eurosystem and deficit banks borrow liquidity directly from the Eurosystem, the Eurosystem assumed the function as an *intermediary* between banks and thereby replaced a bulk of interbank market activities. This intermediary function was reinforced by measures the Eurosystem adopted during the crisis. It narrowed the symmetric corridor that the rates on the facilities form around the MRO-rate and it reduced the requirements that collateral has to fulfill in credit operations. Both measures made transactions with the ECB relatively more attractive than transactions in the interbank market. In our model, the former measure is reflected by an increase in $i^{DF}$ and a decrease in $i^{CF}$. The latter is reflected by a decrease in $\alpha$. The proposition shows that both imply a decrease in $|B|^*$.

### 5.2 Policy Implications

The financial crisis has posed extraordinary challenges to the Eurosystem with regard to its monetary policy as well as with regard to its liquidity management. The primary objective of its *monetary policy* is to maintain price stability, and if it is possible without prejudicing this objective, the Eurosystem is allowed to support the general economic policy of the EU which shall promote, for example, a high level of employment (Treaty establishing the European Community, Article 105). The Eurosystem's *liquidity management* shall ensure that the monetary policy transmission mechanism works properly and that in a financial crisis possible liquidity problems do not result in solvency problems (González-Páramo, 2009). The latter makes clear that during a financial crisis one objective of the Eurosystem's liquidity management is to support the stabilization of the banking sector.

This paper focuses on the Eurosystem's liquidity management. We argue that in the financial crisis significantly increased transaction costs impaired a proper functioning of the interbank market for reserves. An impaired functioning of this market impedes the transmission of monetary policy impulses and may furthermore imply that liquidity problems result in solvency problems. Therefore, the Eurosystem replaced the interbank market by assuming the function as an intermediary between banks. However, this is only a temporary solution, the aim is to reduce this intermediary function and to reactivate the interbank market.

The obvious way to achieve this goal is to reduce transaction costs. However, the high transaction costs on the interbank market for reserves are the result of a high uncertainty about how strongly individual banks are affected by asset losses. Consequently, a reduction in transaction costs cannot be achieved by central bank measures. A possibility for the Eurosystem to reactivate the interbank market is to no longer satisfy total bids in the tender procedure but to allot only the benchmark amount. A further possibility is to make borrowing from the central bank and placing liquidity in the deposit facility less attractive, for example by tightening the criteria which have to be fulfilled by eligible collateral or by expanding the corridor the rates on the deposit and the credit facility perform around the MRO-rate. In our model, this would result in an increase in the parameter $\alpha$ and $i^{CF}$ and in a decrease in $i^{DF}$ respectively, and the results given by regime $b$ in the Proposition reveal that this would lead to an increase in $|B|^*$, i.e. transaction in the interbank market would increase. However, such measures have to be balanced against liquidity problems which may arise and against higher costs for the banking sector. Consequently, the Eurosystem faces a trade-off. On the one hand, it aims at reactivating interbank market activities, on the other hand it aims at supporting the stabilization of the banking sector and the general economic policy of the EU. Therefore, we propose to undertake these measures gradually over time. Over time the uncertainty should decrease so that transaction costs become lower again so that the intermediation function becomes less important.

## 6 Summary

After the collapse of Lehman Brothers in September 2008, there was (1) a strong increase in the banking sector's demand for reserves in the Eurosystem's tender procedures, (2) a strong increase in the use of the Eurosystem's deposit facility, (3) a strong decrease in interbank market transactions, and (4) a systematic fall of the EONIA below the key ECB policy rate. In this paper, we theoretically explain these stylized facts and draw policy implications concerning the Eurosystem's liquidity management. Our model shows that the stylized facts can be explained by a strong increase in transaction costs on the interbank market in combination with the possibility of a nearly unlimited use of central bank credit. The increased transaction costs imply that banks having a liquidity deficit prefer to cover their deficit by borrowing from the central bank rather than in the interbank market. This induces banks with a liquidity surplus to place their excess liquidity in the central bank's deposit facility. Thus, the central bank assumes an intermediary function between banks. The result is an aggregate liquidity surplus in the banking sector which implies a systematic fall of the EONIA below the policy rate. Concerning the implications for the Eurosystem's liquidity management we argue that as long as the interbank market does not function properly, measures to reactivate the interbank market conflict with aims from the monetary policy perspective and the financial stability perspective. Therefore, we propose to undertake these measures gradually over time.

# Appendix

## Proof of Lemma 1

We prove the Lemma by inspecting the properties of the first derivative of $f(B)$ with respect to $B$. Substitution of (4) and (5) in (6) and differentiating yields

$$\frac{\partial f(B)}{\partial B} = \begin{cases} i^{IBM} + \gamma B - (i^{CF} + \alpha) & \text{if} \quad B < A - K \\ i^{IBM} + \gamma B - i^{DF} & \text{if} \quad B > A - K \end{cases} . \tag{23}$$

Note that $f(B)$ is not differentiable at point $B = A - K$. Moreover, note that $\frac{\partial f(B)}{\partial B}$ is increasing in $B$, the limit of $\frac{\partial f(B)}{\partial B}$ as $B$ tends to $(A - K)$ from below satisfies

$$\lim_{B \to (A-K)-} \frac{\partial f(B)}{\partial B} = i^{IBM} + \gamma(A - K) - (i^{CF} + \alpha),$$

and the limit of $\frac{\partial f(B)}{\partial B}$ as $B$ tends to $(A - K)$ from above satisfies

$$\lim_{B \to (A-K)+} \frac{\partial f(B)}{\partial B} = i^{IBM} + \gamma(A - K) - i^{DF}.$$

Accordingly, we need to distinguish three cases:

- Firstly, if $\lim_{B \to (A-K)-} \frac{\partial f(B)}{\partial B} > 0$, it follows from (23) that the optimal transaction $B^{opt}$ in the interbank market is defined by $i^{IBM} + \gamma B^{opt} - (i^{CF} + \alpha) = 0$, implying

$$B^{opt} = \frac{i^{CF} + \alpha - i^{IBM}}{\gamma} \quad \text{if} \quad i^{IBM} + \gamma(A - K) > i^{CF} + \alpha \ . \tag{24}$$

- Secondly, if $\lim_{B \to (A-K)+} \frac{\partial f(B)}{\partial B} < 0$, it follows from (23) that $B^{opt}$ is defined by $i^{IBM} + \gamma B^{opt} - i^{DF} = 0$, implying

$$B^{opt} = -\frac{i^{IBM} - i^{DF}}{\gamma} \quad \text{if} \quad i^{IBM} + \gamma(A - K) < i^{DF} \ . \tag{25}$$

- Thirdly, if $\lim_{B \to (A-K)-} \leq 0$ and $\lim_{B \to (A-K)+} \geq 0$, it follows from (23) that:

$$B^{opt} = A - K \quad \text{if} \quad i^{IBM} + \gamma(A - K) \in [i^{DF}, i^{CF} + \alpha] \ . \tag{26}$$

Restricting attention to $i^{IBM} \in [i^{DF}, i^{CF} + \alpha]$ and distinguishing between the case of a bank with a liquidity gap $(A - K > 0)$ an a bank with excess liquidity $(K - A \geq 0)$, (24), (25) and (26) directly result in (7) and (8). $\blacksquare$

## Proof of Lemma 2

We prove the Lemma by inspecting the properties of the market clearing condition $B^{a^{opt}} + B^{b^{opt}} = 0$. Restricting attention to $i^{IBM} \in [i^{DF}, i^{CF} + \alpha]$, we proceed in two steps:

- Firstly, suppose that $K^a - A^a \geq 0$ (and $K^b - A^b > 0$). Then, substitution of (8) in the market clearing condition $B^{a^{opt}} + B^{b^{opt}} = 0$ gives

  $$-\min\left\{K^a - A^a, \frac{i^{IBM} - i^{DF}}{\gamma}\right\} - \min\left\{K^b - A^b, \frac{i^{IBM} - i^{DF}}{\gamma}\right\} = 0.$$

  This condition is met only if $i^{IBM} = i^{DF}$; substitution of this in (8) gives $|B^{opt}| = 0$. Denoting these equilibrium values by $i^{IBM^*}$ and $|B|^*$ respectively, we obtain (10) and (11) in Lemma 2.

- Secondly, suppose that $A^a - K^a > 0$ (and $K^b - A^b > 0$). Then, substitution of (7) and (8) in the market clearing condition $B^{a^{opt}} + B^{b^{opt}} = 0$ gives

  $$\min\left\{A^a - K^a, \frac{i^{CF} + \alpha - i^{IBM}}{\gamma}\right\} - \min\left\{K^b - A^b, \frac{i^{IBM} - i^{DF}}{\gamma}\right\} = 0. \tag{27}$$

  Now, we can distinguish three subcases:

  - Firstly, suppose that $A^a - K^a \in ]0, K^b - A^b[$. Then, if $i^{IBM} > i^{DF} + \gamma(A^a - K^a)$ and thus $\min\left\{K^b - A^b, \frac{i^{IBM} - i^{DF}}{\gamma}\right\} > A^a - K^a$, the LHS of (27) is smaller than 0. Accordingly, we can restrict attention to $i^{IBM} \leq i^{DF} + \gamma(A^a - K^a)$ so that (27) becomes

    $$\min\left\{A^a - K^a, \frac{i^{CF} + \alpha - i^{IBM}}{\gamma}\right\} - \frac{i^{IBM} - i^{DF}}{\gamma} = 0. \tag{28}$$

    This condition is met if $i^{IBM} = i^{DF} + \gamma \min\{A^a - K^a, |B|^{marg}\}$; substitution of this in (7) or (8) gives $|B^{opt}| = \min\{A^a - K^a, |B|^{marg}\}$. Denoting these equilibrium values by $i^{IBM^*}$ and $|B|^*$ respectively, we obtain (12) and (13) in Lemma 2. As the LHS of (28) is strictly decreasing in $i^{IBM}$, this equilibrium is unique.

– Secondly, suppose that $A^a - K^a > K^b - A^b$. Then, if $i^{IBM} < i^{CF} + \alpha - \gamma(K^b - A^b)$ and thus $\min\left\{A^a - K^a, \frac{i^{CF}+\alpha-i^{IBM}}{\gamma}\right\} > K^b - A^b$, the LHS of (27) is bigger than 0. Accordingly, we can restrict attention to $i^{IBM} \geq i^{CF} + \alpha - \gamma(K^b - A^b)$ so that (27) becomes

$$\frac{i^{CF} + \alpha - i^{IBM}}{\gamma} - \min\left\{K^b - A^b, \frac{i^{IBM} - i^{DF}}{\gamma}\right\} = 0. \tag{29}$$

This condition is met if $i^{IBM} = i^{CF} + \alpha - \gamma \min\left\{K^b - A^b, |B|^{marg}\right\}$; substitution of this in (7) or (8) gives $|B^{opt}| = \min\left\{K^b - A^b, |B|^{marg}\right\}$. Denoting these equilibrium values by $i^{IBM^*}$ and $|B|^*$ respectively, we obtain (14) and (15) in Lemma 2. As the LHS of (29) is strictly decreasing in $i^{IBM}$, this equilibrium is unique.

– Thirdly, suppose that $A^a - K^a = K^b - A^b$. By parallel arguments as above, it can be shown that in this case, (27) is met only if $i^{IBM} \in \left[i^{DF} + \gamma \min\left\{K^b - A^b, |B|^{marg}\right\}, i^{CF} + \alpha - \gamma \min\left\{K^b - A^b, |B|^{marg}\right\}\right]$; substitution of this in (7) or (8) gives $|B^{opt}| = \min\left\{K^b - A^b, |B|^{marg}\right\}$. Denoting these equilibrium values by $i^{IBM^*}$ and $|B|^*$ respectively, we obtain (16) and (17) in Lemma 2. ■

## Proof of the Proposition

We prove the Proposition in two steps. We first investigate the bidding incentives of each individual bank in the MRO. Then, we derive the overall equilibrium. Firstly, consider an individual bank which aims at minimizing $g(K)$. Substitution of (1), (4) and (5) in (18), together with Lemma 1, gives

$$\begin{aligned} g(K) &= \left(i^{MRO} + \alpha\right) K + \frac{1}{2}\left[i_L^{IBM} B_L^{opt} + \frac{1}{2}\gamma(B_L^{opt})^2\right] + \frac{1}{2}\left[i_H^{IBM} B_H^{opt} + \frac{1}{2}\gamma(B_H^{opt})^2\right] \\ &+ \begin{cases} \frac{1}{2}\left(i^{CF} + \alpha\right)\left(A_L - K - B_L^{opt}\right) \\ +\frac{1}{2}\left(i^{CF} + \alpha\right)\left(A_H - K - B_H^{opt}\right) & \text{if} \quad K < A_L \\ -\frac{1}{2} i^{DF}\left(B_L^{opt} + K - A_L\right) \\ +\frac{1}{2}\left(i^{CF} + \alpha\right)\left(A_H - K - B_H^{opt}\right) & \text{if} \quad K \in [A_L, A_H[ \\ -\frac{1}{2} i^{DF}\left(B_L^{opt} + K - A_L\right) \\ -\frac{1}{2} i^{DF}\left(B_H^{opt} + K - A_H\right) & \text{if} \quad K \geq A_H \end{cases} \end{aligned}$$

and thus

$$\frac{\partial g(K)}{\partial K} = i^{MRO} + \alpha$$
$$+ \begin{cases} \frac{1}{2}\left[i_L^{IBM} + \gamma B_L^{opt} - (i^{CF} + \alpha)\right] \frac{\partial B_L^{opt}}{\partial K} - \frac{1}{2}(i^{CF} + \alpha) \\ +\frac{1}{2}\left[i_H^{IBM} + \gamma B_H^{opt} - (i^{CF} + \alpha)\right] \frac{\partial B_H^{opt}}{\partial K} - \frac{1}{2}(i^{CF} + \alpha) & \text{if} \quad K < A_L \\ \frac{1}{2}\left[i_L^{IBM} + \gamma B_L^{opt} - i^{DF}\right] \frac{\partial B_L^{opt}}{\partial K} - \frac{1}{2}i^{DF} \\ +\frac{1}{2}\left[i_H^{IBM} + \gamma B_H^{opt} - (i^{CF} + \alpha)\right] \frac{\partial B_H^{opt}}{\partial K} - \frac{1}{2}(i^{CF} + \alpha) & \text{if} \quad K \in [A_L, A_H[ \\ \frac{1}{2}\left[i_L^{IBM} + \gamma B_L^{opt} - i^{DF}\right] \frac{\partial B_L^{opt}}{\partial K} - \frac{1}{2}i^{DF} \\ +\frac{1}{2}\left[i_H^{IBM} + \gamma B_H^{opt} - i^{DF}\right] \frac{\partial B_H^{opt}}{\partial K} - \frac{1}{2}i^{DF} & \text{if} \quad K \geq A_H \end{cases} \tag{30}$$

For a given liquidity need $A_i$ with $i = L, H$, it follows from (7) that if $K < A_i$, we have

$$\frac{\partial B_i^{opt}}{\partial K} = \begin{cases} -1 & \text{if} \quad i_i^{IBM} + \gamma(A_i - K) - (i^{CF} + \alpha) \leq 0 \\ 0 & \text{if} \quad i_i^{IBM} + \gamma(A_i - K) - (i^{CF} + \alpha) > 0 \end{cases}, \tag{31}$$

while it follows from (8) that if $K \geq A_i$, we have:

$$\frac{\partial B_i^{opt}}{\partial K} = \begin{cases} 0 & \text{if} \quad i_i^{IBM} + \gamma(A_i - K) - i^{DF} < 0 \\ -1 & \text{if} \quad i_i^{IBM} + \gamma(A_i - K) - i^{DF} \geq 0 \end{cases}. \tag{32}$$

Substitution of (7), (8), (31) and (32) in (30) gives

$$\frac{\partial g(K)}{\partial K} = i^{MRO} + \alpha$$
$$- \begin{cases} \frac{1}{2}\min\left\{i_L^{IBM} + \gamma(A_L - K), i^{CF} + \alpha\right\} \\ -\frac{1}{2}\min\left\{i_H^{IBM} + \gamma(A_H - K), i^{CF} + \alpha\right\} & \text{if} \quad K < A_L \\ \frac{1}{2}\max\left\{i_L^{IBM} + \gamma(A_L - K), i^{DF}\right\} \\ -\frac{1}{2}\min\left\{i_H^{IBM} + \gamma(A_H - K), i^{CF} + \alpha\right\} & \text{if} \quad K \in [A_L, A_H[ \\ \frac{1}{2}\max\left\{i_L^{IBM} + \gamma(A_L - K), i^{DF}\right\} \\ -\frac{1}{2}\max\left\{i_H^{IBM} + \gamma(A_H - K), i^{DF}\right\} & \text{if} \quad K \geq A_H \end{cases}. \tag{33}$$

Note that for $i^{IBM} \in [i^{DF}, i^{CF} + \alpha]$ (which is true as Lemma 2 indicates), $\frac{\partial g(K)}{\partial K}$ is (weakly) increasing in $K$. From this, we can already derive three preliminary results with respect to the overall equilibrium:

- Firstly, an equilibrium with $K^{a^*} \geq A^a_L$ is feasible only if $\frac{\partial g(K^a)}{\partial K^a} \mid_{A^a_L} \leq 0$. This condition, however, cannot be met. To see this, note that for bank $a$, which has a liquidity need $A^a_L$ in state 1 and $A^a_H$ in state 2, (33) implies

$$\frac{\partial g(K^a)}{\partial K^a} \mid_{A^a_L} = i^{MRO} + \alpha - \frac{1}{2} i_1^{IBM} - \frac{1}{2} \min \left\{ i_2^{IBM} + \gamma(A^a_H - A^a_L), i^{CF} + \alpha \right\}. \quad (34)$$

Now, note that $i^{MRO} = \frac{i^{CF} + i^{DF}}{2}$ and that in the case of $K^{a^*} \geq A^a_L$, (10) implies $i_1^{IBM*} = i^{DF}$. Therefore, (34) can be modified to

$$\frac{\partial g(K^a)}{\partial K^a} \mid_{A^a_L} = \frac{1}{2} i^{CF} + \alpha - \frac{1}{2} \min \left\{ i_2^{IBM} + \gamma(A^a_H - A^a_L), i^{CF} + \alpha \right\} \geq \frac{1}{2}\alpha,$$

so that we can conclude that there is no equilibrium in which bank $a$ makes a bid $K^{a^*} \geq A^a_L$.

- Secondly, an equilibrium with $K^{a^*} < A^a_L$ and $K^{b^*} > 0$ is feasible only if $\frac{\partial g(K^a)}{\partial K^a} \mid_{K^{a^*} < A^a_L} \geq 0$ and $\frac{\partial g(K^b)}{\partial K^b} \mid_{K^{b^*} > 0} = 0$. These conditions, however, cannot be met simultaneously. To see this, note first that for bank $a$, (33) implies that $\frac{\partial g(K^a)}{\partial K^a} \mid_{K^{a^*} < A^a_L} \geq 0$ is met only if

$$\begin{aligned} i^{MRO} + \alpha \quad &\geq \quad \frac{1}{2} \min \left\{ i_1^{IBM} + \gamma(A^a_L - K^{a^*}), i^{CF} + \alpha \right\} \\ &+ \quad \frac{1}{2} \min \left\{ i_2^{IBM} + \gamma(A^a_H - K^{a^*}), i^{CF} + \alpha \right\}, \end{aligned} \quad (35)$$

while for bank $b$, which has a liquidity need $A^b_H$ in state 1 and $A^b_L$ in state 2, (33) implies that $\frac{\partial g(K^b)}{\partial K^b} \mid_{K^{b^*} > 0} = 0$ is met only if

$$\begin{aligned} i^{MRO} + \alpha \quad &= \quad \frac{1}{2} \max \left\{ i_2^{IBM} + \gamma(A^b_L - K^{b^*}), i^{DF} \right\} \\ &+ \quad \frac{1}{2} \max \left\{ i_1^{IBM} + \gamma(A^b_H - K^{b^*}), i^{DF} \right\}. \end{aligned} \quad (36)$$

Note that (35) and (36) could be met simultaneously only if the RHS of (36) were not smaller than the RHS of (35). However, since $A^b_L < A^b_H < 0$, the RHS of (36) is equal to $i^{DF}$ for $i_1^{IBM} = i_2^{IBM} = i^{DF}$ and strictly smaller than $\frac{1}{2}(i_1^{IBM} + i_2^{IBM})$ otherwise. Moreover, since $K^{a^*} < A^a_L < A^a_H$, the RHS of (35) is equal to $i^{CF} + \alpha$ for $i_1^{IBM} = i_2^{IBM} = i^{CF} + \alpha$ and strictly larger than $\frac{1}{2}(i_1^{IBM} + i_2^{IBM})$ otherwise.

Therefore, (35) and (36) cannot be met simultaneously and we can conclude that there is no equilibrium with $K^{a^*} < A_L^a$ and $K^{b^*} > 0$.

- Thirdly, an equilibrium with $K^{a^*} < D := A_L^a + A_H^b = A_H^a + A_L^b$ and $K^{b^*} = 0$ is feasible only if $\frac{\partial g(K^a)}{\partial K^a} \mid_{K^{a*}<D} \geq 0$. This condition, however, cannot be met. To see this, note that for bank $a$, (33) implies

$$\begin{aligned} \frac{\partial g(K^a)}{\partial K^a} \mid_{K^{a*}<D} &= i^{MRO} + \alpha - \frac{1}{2}\min\left\{i_1^{IBM} + \gamma(A_L^a - K^{a^*}), i^{CF} + \alpha\right\} \qquad (37) \\ &- \frac{1}{2}\min\left\{i_2^{IBM} + \gamma(A_H^a - K^{a^*}), i^{CF} + \alpha\right\}. \end{aligned}$$

Now, note that if $K^{a^*} < D$ and $K^{b^*} = 0$, it follows from (14) and (15) that

$$i_1^{IBM^*} > i^{CF} + \alpha - \gamma(A_L^a - K^{a^*}), \qquad (38)$$

$$i_2^{IBM^*} > i^{CF} + \alpha - \gamma(A_H^a - K^{a^*}). \qquad (39)$$

Therefore (37) can be modified to

$$\frac{\partial g(K^a)}{\partial K^a} \mid_{K^{a*}<D} = i^{MRO} + \alpha - (i^{CF} + \alpha) < 0,$$

so that we can conclude that there is no equilibrium with $K^{a^*} < D$ and $K^{b^*} = 0$.

The three preliminary results imply that we can restrict attention to equilibria with $K^{a^*} \in [D, A_L^a[$ and $K^{b^*} = 0$, which are feasible only if $\frac{\partial g(K^a)}{\partial K^a} \mid_{K^{a*} \in [D, A_L^a[} = 0$ and $\frac{\partial g(K^b)}{\partial K^b} \mid_{0 > A_H^b} \geq 0$. Now, note that (33) implies for bank $a$ that $\frac{\partial g(K^a)}{\partial K^a} \mid_{K^{a*} \in [D, A_L^a[} = 0$ is met only if

$$\begin{aligned} i^{MRO} + \alpha &= \frac{1}{2}\min\left\{i_1^{IBM} + \gamma(A_L^a - K^{a^*}), i^{CF} + \alpha\right\} \qquad (40) \\ &+ \frac{1}{2}\min\left\{i_2^{IBM} + \gamma(A_H^a - K^{a^*}), i^{CF} + \alpha\right\}, \end{aligned}$$

while (33) implies for bank $b$ that $\frac{\partial g(K^b)}{\partial K^b} \mid_{0 > A_H^b} \geq 0$ is met only if

$$i^{MRO} + \alpha \geq \frac{1}{2}\max\left\{i_2^{IBM} + \gamma A_L^b, i^{DF}\right\} + \frac{1}{2}\max\left\{i_1^{IBM} + \gamma A_H^b, i^{DF}\right\}. \qquad (41)$$

Now, it is useful to distinguish two cases:

- Firstly, consider an equilibrium with $K^{a^*} = D$ and $K^{b^*} = 0$. In this case, (40) can be rewritten to

$$\begin{aligned} i^{MRO} + \alpha &= \frac{1}{2}\min\left\{i_1^{IBM} + \gamma(A_L^a - D), i^{CF} + \alpha\right\} \\ &+ \frac{1}{2}\min\left\{i_2^{IBM} + \gamma(A_H^a - D), i^{CF} + \alpha\right\}, \end{aligned} \tag{42}$$

and, since $A_L^b = D - A_H^a$ and $A_H^b = D - A_L^a$, (41) can be rewritten to

$$\begin{aligned} i^{MRO} + \alpha &\geq \frac{1}{2}\max\left\{i_2^{IBM} + \gamma(D - A_H^a), i^{DF}\right\} \\ &+ \frac{1}{2}\max\left\{i_1^{IBM} + \gamma(D - A_L^a), i^{DF}\right\}. \end{aligned} \tag{43}$$

Since $A_H^a > A_L^a > D$, the RHS of (42) is larger than the RHS of (43). Accordingly, (43) will be met if (42) is met so that we can restrict attention to (42). Now, we can consider two subcases:

  – Firstly, suppose that $A_H^a - D \leq |B|^{marg}$ (and thus $\gamma \leq \frac{\frac{1}{2}(i^{CF}+\alpha-i^{DF})}{A_H^a - D} =: \widehat{\gamma}$). Then, it follows from (16) and (17) that

$$i_1^{IBM^*} \in [i^{DF} + \gamma(A_L^a - D), i^{CF} + \alpha - \gamma(A_L^a - D)], \tag{44}$$

$$i_2^{IBM^*} \in [i^{DF} + \gamma(A_H^a - D), i^{CF} + \alpha - \gamma(A_H^a - D)], \tag{45}$$

  so that we can rewrite (42) to

$$i^{MRO} + \alpha = \frac{1}{2}[i_1^{IBM} + \gamma(A_L^a - D)] + \frac{1}{2}[i_2^{IBM} + \gamma(A_H^a - D)],$$

  implying

$$E[i^{IBM}] := \frac{1}{2}i_1^{IBM} + \frac{1}{2}i_2^{IBM} = i^{MRO} + \alpha - \gamma(E[A^a] - D).$$

  This is consistent with (44) and (45) only if

$$i^{MRO} + \alpha - \gamma(E[A^a] - D) \geq i^{DF} + \gamma(E[A^a] - D),$$

  implying $\gamma \leq \frac{\frac{1}{2}(i^{MRO}+\alpha-i^{DF})}{E[A^a]-D} =: \bar{\gamma}$. To sum up this subcase, $K^{a^*} = D$ and $K^{b^*} = 0$ is an equilibrium if $\gamma \leq \min\{\widehat{\gamma}, \bar{\gamma}\}$.

– Secondly, suppose that $A_H^a - D > |B|^{marg} \geq A_L^a - D$ (and thus $\gamma \in ]\widehat{\gamma}, \frac{\frac{1}{2}(i^{CF}+\alpha-i^{DF})}{A_L^a-D}]$). Then, it follows from (16) and (17) that

$$\begin{aligned} i_1^{IBM*} &\in [i^{DF} + \gamma(A_L^a - D), i^{CF} + \alpha - \gamma(A_L^a - D)], & (46)\\ i_2^{IBM*} &= \frac{1}{2}(i^{CF} + \alpha + i^{DF}), & (47) \end{aligned}$$

so that we can rewrite (42) to

$$i^{MRO} + \alpha = \frac{1}{2}[i_1^{IBM} + \gamma(A_L^a - D)] + \frac{1}{2}(i^{CF} + \alpha),$$

implying

$$i_1^{IBM} = i^{DF} + \alpha - \gamma(A_L^a - D).$$

This is consistent with (46) only if

$$i^{DF} + \alpha - \gamma(A_L^a - D) \geq i^{DF} + \gamma(A_L^a - D)$$

implying $\gamma \leq \frac{\frac{1}{2}\alpha}{A_L^a - D} =: \widetilde{\gamma}$. To sum up this subcase, $K^{a^*} = D$ and $K^{b^*} = 0$ is an equilibrium if $\gamma \in ]\widehat{\gamma}, \widetilde{\gamma}]$.

- Secondly, consider an equilibrium with $K^{a^*} \in ]D, A_L^a[$ and $K^{b^*} = 0$. In this case, it follows from (12) and (13) that

$$\begin{aligned} i_1^{IBM*} &= i^{DF} + \gamma \min\left\{A_L^a - K^{a^*}, |B|^{marg}\right\}, & (48)\\ i_2^{IBM*} &= i^{DF} + \gamma \min\left\{A_H^a - K^{a^*}, |B|^{marg}\right\}. & (49) \end{aligned}$$

Substitution of (48) and (49) in (40) yields

$$\begin{aligned} i^{MRO} + \alpha &= \frac{1}{2}\min\left\{i^{DF} + 2\gamma(A_L^a - K^{a^*}), i^{CF} + \alpha\right\} & (50)\\ &+ \frac{1}{2}\min\left\{i^{DF} + 2\gamma(A_H^a - K^{a^*}), i^{CF} + \alpha\right\} \end{aligned}$$

and substitution of (48) and (49) in (41) yields $i^{MRO} + \alpha \geq i^{DF}$. As this condition is always met, we can restrict attention to (50). Now, note that (50) can be rearranged to

$$K^{a^*} = \begin{cases} E[A^a] - \frac{\frac{1}{2}(i^{MRO}+\alpha-i^{DF})}{\gamma} & \text{if} \quad K^{a^*} \geq A^a_H - \frac{\frac{1}{2}(i^{CF}+\alpha-i^{DF})}{\gamma} \\ A^a_L - \frac{\frac{1}{2}\alpha}{\gamma} & \text{if} \quad K^{a^*} < A^a_H - \frac{\frac{1}{2}(i^{CF}+\alpha-i^{DF})}{\gamma} \end{cases}$$

implying

$$K^{a^*} = \begin{cases} E[A^a] - \frac{\frac{1}{2}(i^{MRO}+\alpha-i^{DF})}{\gamma} & \text{if} \quad \gamma \leq \frac{\frac{1}{2}(i^{CF}-i^{DF})}{A^a_H - A^a_L} =: \bar{\bar{\gamma}} \\ A^a_L - \frac{\frac{1}{2}\alpha}{\gamma} & \text{if} \quad \gamma > \frac{\frac{1}{2}(i^{CF}-i^{DF})}{A^a_H - A^a_L} =: \bar{\bar{\gamma}} \end{cases}.$$

Due to the requirement $K^{a^*} > D$, we can sum up this subcase by stating that there is an equilibrium with

$$K^{a^*} = \begin{cases} E[A^a] - \frac{\frac{1}{2}(i^{MRO}+\alpha-i^{DF})}{\gamma} & \text{if} \quad \gamma \in ]\widetilde{\gamma}, \bar{\bar{\gamma}}] \\ A^a_L - \frac{\frac{1}{2}\alpha}{\gamma} & \text{if} \quad \gamma > \max\{\bar{\bar{\gamma}}, \widetilde{\gamma}\} \end{cases} \tag{51}$$

and $K^{b^*} = 0$.

Note that only if $\bar{\gamma} < \widehat{\gamma}$, we have $\bar{\gamma} < \bar{\bar{\gamma}}$, $\widetilde{\gamma} < \bar{\bar{\gamma}}$ and $\widetilde{\gamma} < \widehat{\gamma}$. Therefore, we obtain:

- If $\bar{\gamma} < \widehat{\gamma}$, there is an equilibrium with $K^{b^*} = 0$ and

$$K^{a^*} = \begin{cases} D & \text{if} \quad \gamma \leq \bar{\gamma} \\ E[A^a] - \frac{\frac{1}{2}(i^{MRO}+\alpha-i^{DF})}{\gamma} & \text{if} \quad \gamma \in ]\bar{\gamma}, \bar{\bar{\gamma}}] \\ A^a_L - \frac{\frac{1}{2}\alpha}{\gamma} & \text{if} \quad \gamma > \bar{\bar{\gamma}} \end{cases}, \tag{52}$$

  The remaining variables stated in the proposition then can be found by substituting (52) and $K^{b^*} = 0$ in Lemma 1 and 2 and by keeping in mind that $DF = \max\left\{K^{b^*} - |B|^* - A^b, 0\right\}$ and $CF = \max\left\{A^a - K^{a^*} - |B|^*, 0\right\}$.

- If $\bar{\gamma} \geq \widehat{\gamma}$, there is an equilibrium with $K^{b^*} = 0$ and

$$K^{a^*} = \begin{cases} D & \text{if} \quad \gamma \leq \widetilde{\gamma} \\ A^a_L - \frac{\frac{1}{2}\alpha}{\gamma} & \text{if} \quad \gamma > \widetilde{\gamma} \end{cases}. \tag{53}$$

  As this equilibrium does not fit to the stylized facts observed in the euro area, we have abstained from presenting it formally in the Proposition. ■

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