Interbank Market Frictions - Implications for Bank Loan Supply and Monetary Policy*

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Abstract

We analyze the impact of overnight interbank market frictions on bank loan supply when banks face idiosyncratic liquidity risk and discuss resulting implications for monetary policy implementation. Both, frictions and risk, negatively influence bank loan supply. However, by means of its standing facilities, the central bank not only offers an alternative to using the interbank market but also determines the costs of friction-induced holdings of positive or negative precautionary liquidity. Therefore, the facilities allow the central bank to influence banks' expected liquidity costs, and thereby their loan supply, so that interbank market frictions need not be an impediment to monetary policy transmission.

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1 Introduction

The overnight interbank market is the starting point of the monetary policy transmission mechanism. By steering the interest rate in this market, the central bank aims to affect banks' funding costs in order to influence bank loan supply and thereby aggregate demand. In the recent past, especially during the peaks of the financial crisis, we witnessed the seizing up of the overnight interbank market.[1] For various reasons, trading in this market became more expensive. Search costs increased as finding counterparties with matching liquidity needs became more difficult due, for example, to reduced credit lines among banks.[2] Also, asymmetric information costs rose as uncertainty about the credit risk of interbank lending increased so that banks intensified their costly creditworthiness checks of potential borrowers, who in turn increased the costly signaling of their creditworthiness.[3] Another potential source of higher interbank trading costs may be found in new financial regulations.[4] Due to the importance of the overnight interbank market for monetary policy implementation, the observed stress in this market raised concerns about the central bank's ability to actually influence bank loan supply and triggered a heated debate on the potential impairment of the transmission mechanism of monetary policy and the consequences for monetary policy implementation.

Our paper aims to contribute to this debate by analyzing the impact of overnight interbank market frictions on bank loan supply when banks face idiosyncratic liquidity risk and by discussing resulting implications for monetary policy implementation. We develop a theoretical model in which banks have to decide on their loan supply to the non-banking sector in the presence of bank idiosyncratic liquidity risk and interbank market frictions. We capture the frictions in the form of broadly defined transaction costs. These costs may

[1] For a recent documentation on stress in the overnight interbank market in the euro area over the course of the financial and sovereign debt crisis in Europe see, for example, Frutos et al. (2016).

[2] There are a couple of recently published papers that have developed interbank market models dealing with the search for suitable counterparties and bilateral negotiations. See, for example, Afonso and Lagos (2015), Bech and Monnet (2016), and Vollmer and Wiese (2016).

[3] Consequences of asymmetric information in interbank markets are analyzed, for example, in Freixas and Jorge (2008) and Heider et al. (2015).

[4] For a recent discussion on the influence of new financial regulations on interbank market conditions see, for example, Bech and Keister (2013), Jackson and Noss (2015), Bank for International Settlements, Committee on the Gloabl Financial System (2015), and Bindseil (2016).

include search costs, asymmetric information costs and regulatory costs, i.e., similar to Bartolini et al. (2001), we argue that frictions make interbank trading more expensive.[5]

We find that both, interbank market frictions and bank idiosyncratic liquidity risk, negatively influence bank loan supply. However, they do not impede the transmission mechanism of monetary policy. By means of its standing facilities, the central bank is able to influence banks' expected liquidity costs, and thus their loan supply, also in the presence of significant, even prohibitive interbank market transaction costs. The reason is twofold. First, with its standing facilities the central bank provides an alternative to using the costly interbank market.[6] Second, by setting the rates on its facilities the central bank determines the costs and therefore the amount of friction-induced holdings of positive or negative precautionary liquidity, and the more precautionary liquidity (in absolute terms) banks hold, the lower their expected liquidity costs are.

Our model allows us to draw some conclusions for the ECB's monetary policy and bank loan supply in the euro area. From the beginning of the recent financial crisis up to now (May 2017) the ECB has not only significantly reduced banks' expected liquidity costs by lowering its key interest rates but also by changing the rates on its standing facilities in a way that interbank market friction-induced liquidity costs have become extremely low. The ECB's monetary policy has thus impeded interbank market frictions to become an impediment for the transmission mechanism of monetary policy. Consequently, these fictions have not been responsible for weak bank lending in the euro area, especially in the periphery.

In our theoretical analysis, banks' holdings of precautionary liquidity thus play a crucial role. Banks will hold precautionary liquidity if interbank market frictions are sufficiently

[5]In our model, there is not an adverse section problem due to asymmetric information as this would mean that lending to the overnight interbank market involves credit risk. However, in the *overnight* interbank market, lenders are typically unwilling to expose themselves to any counterparty credit risk as even for the slightest risk, the adequate interest rate is much higher than the rate on the alternative liquidity source, the central bank's lending facility (see Hauck and Neyer (2014) for a numerical example). We thus assume that asymmetric information leads to costly creditworthiness checks and costly signaling to eliminate the asymmetries, so that those overnight interbank market transactions actually taking place are risk-free.

[6]If banks make use of this alternative, the central bank will become an intermediary between liquidity deficit and surplus banks, i.e., interbank market activities will decline. In this context Bindseil (2016), among others, points to the trade-off between the central bank's aim to control the interbank rate (narrow interest corridor) and to maintain market activities (wide interest corridor). In the same vein, in our model a trade-off between boosting bank loan supply and maintaining interbank market activity could be discussed. However, our paper does not broach this trade-off but focuses on the impact of monetary policy measures on bank loan supply.

strong and if the expected costs of a liquidity deficit and of a liquidity surplus are asymmetric. Frictions will be sufficiently strong if they make interbank trading so expensive that at least one market side prefers to use the central bank's facilities, unless the other market side is willing to bear a higher share of the transaction costs. The asymmetric expected costs of a deficit and of a surplus may result from different probabilities of facing a deficit/surplus, stigma or regulatory costs which are relevant only to one interbank market side, or an asymmetric interest corridor formed by the rates on the central bank's standing facilities around the main policy rate. If banks expect a deficit to be more expensive than a surplus, they will hold positive precautionary liquidity, i.e., more liquidity than they expect to need, otherwise they will hold negative precautionary liquidity.

Our model captures main elements of the Eurosystem's operational framework, the main refinancing operations, and its two standing facilities.[7] Nevertheless, our main results also apply to other central banks' operational frameworks. Crucial is that banks facing stochastic deposit flows are induced to use the central bank's facilities to balance their actual liquidity position.

The rest of this paper is organized as follows. Section 2 presents related literature. Section 3 describes the framework of the model. Section 4 derives the optimal liquidity management and optimal loan supply of an individual bank. Differing in the extent of interbank market frictions, three possible aggregate equilibria are presented in Section 5. Section 6 discusses, for each possible equilibrium, the impact of different monetary policy measures on bank loan supply. Section 7 concludes the paper.

2 Related Literature

Our paper contributes to three strands of literature. The first strand analyzes how far frictions in financial markets influence the impact of monetary policy on bank lending. A huge part of this literature considers asymmetric information in credit markets and argues

[7] The main refinancing operations are credit operations with a maturity of one week by which the Eurosystem provides reserves to the banking sector. The two standing facilities, a deposit facility and a lending facility, allow banks to balance their overnight liquidity needs. The interest rates on the facilities form a corridor around the rate on the main refinancing operations with a deposit rate that is lower and a lending rate that is higher than the main refinancing rate. For a detailed description of the Eurosystem's operational framework see European Central Bank (2012a) and the ECB's website.

that these frictions amplify the effects of monetary policy on bank lending and therefore on aggregate demand.[8]

The second strand of related literature deals with interbank market frictions. Until the outbreak of the financial crisis in 2007, the overnight interbank market was typically regarded as frictionless. As a result, papers dealing with this issue were rather rare. One of these papers is by Bartolini et al. (2001). They consider interbank market transaction costs and show that these costs are responsible for a systematically high interbank rate at the end of reserve maintenance periods. However, the financial crisis inspired a rapidly growing literature dealing with interbank market imperfections. Freixas and Jorge (2008) show that asymmetric information about credit risk in the interbank market may induce the rationing of firms in credit markets, which leads first to a large impact of monetary policy on aggregate investment given the small interest elasticity of investment and second to a stronger impact on banks with less liquid balance sheets. Heider et al. (2015) argue that asymmetric information about credit risk in the interbank market induces banks to hoard liquidity which may result in either adverse selection or a dry-up in the interbank market. However, banks may learn about counterparty credit risks by repeatedly trading with each other so that the asymmetric information problem may be mitigated. In an empirical analysis of the German unsecured overnight interbank market, Bräuning and Fecht (2017) determine the impact of such relationship lending on banks' ability to access liquidity. Iyer et al. (2014) show that the contractionary effect of the impaired functioning of the Portuguese interbank market during the crisis on bank loan supply was stronger with banks which had less relationship lending. Besides Heider et al. (2015), there are further papers that analyze how far interbank market frictions induce banks to hold precautionary liquidity. Ashcraft et al. (2011) show that credit constraints and limited participation in combination with possible payment shocks can explain the precautionary holding of liquidity. Referring to the term interbank market, Acharya and

[8]For a survey on this credit view of monetary policy see, for example, Boivin et al. (2010) and Peek and Rosengreen (2012). One part of this credit view of monetary policy, the balance sheet channel, focuses on the impact of monetary policy on borrowers' net worth. Seminal papers dealing with the balance sheet channel are Bernanke and Gertler (1989) and Bernanke et al. (1999). The other part of the credit view, the bank lending channel, focuses on the impact on bank deposits. Important papers dealing with this traditional bank lending channel are Gertler and Gilchrist (1993) and Kashyap and Stein (1995). Disyatat (2011) shows that a greater reliance on market-based funding instead of deposits creates a new approach to the bank lending channel.

Skeie (2011) trace precautionary liquidity holdings of potential lending banks back to the rollover risk of their own debt. Considering the overnight interbank market in the UK, Acharya and Merrouche (2012) argue that impaired markets for wholesale funding also lead to precautionary holding of liquidity. Allen et al. (2009) show that if uncertainty about aggregate liquidity demand compared to idiosyncratic demand is sufficiently high, banks will start to hoard liquidity.

The third strand of related literature deals with the interbank market and monetary policy implementation.[9] For monetary policy implementation the overnight interbank market plays a crucial role. The seminal paper in this literature is by Poole (1968). He develops a stochastic bank reserve management model in which banks are subject to a late payment shock, i.e., a liquidity shock which occurs after the closure of the interbank market. Banks ending the day with a liquidity deficit have to borrow from the central bank at a rate that is high relative to the interbank rate. Those banks ending with a liquidity surplus face forgone interest revenues from lending to the interbank market. Under uncertainty the banks, aiming to maximize their expected profit, thus have to decide on their optimal interbank market transactions. Poole's idea of using a late payment shock to introduce uncertainty into a bank reserve management model has been taken up by lots of papers.[10] Generally, prior to the outbreak of the financial crisis in 2007, the literature on monetary policy implementation focuses on specific institutional aspects to explain interbank market participant behavior and determinants of the interbank rate. Ho and Saunders (1985) and Clouse and Dow (2002), for example, consider major institutional characteristics of the US federal funds market. With respect to the federal funds market, a huge part of the literature analyzes why the federal funds rate fails to follow a martingale within the reserve maintenance period.[11] A bulk of the literature on monetary policy implementation in the euro area deals with the under- and overbidding behavior in the Eurosystem's main refinancing operations which could be observed

[9]For a survey see, for example, Friedman and Kuttner (2011). By describing and discussing different parts of a central bank's operational framework, Bindseil (2014) gives a broad survey of monetary policy implementation in times of non-crisis and crisis.

[10]See, for example, Furfine (2000), Bindseil et al. (2006), Whitesell (2006), and Bech and Monnet (2016).

[11]See, for example, Campbell (1987), Hamilton (1996), Clouse and Dow (1999), Furfine (2000), and Bartolini et al. (2001, 2002).

in the first years of the European Monetary Union.[12] Apart from this, there are papers that examine the consequences of alternative monetary policy implementation measures. Nautz (1998) shows that the central bank can influence the interbank market rate by being more or less vague about its future monetary policy. Välimäki (2001) analyzes the effects of alternative tender procedures with respect to the Eurosystem's refinancing operations. Neyer and Wiemers (2004) refer to the collateral framework. They show that differences in banks' opportunity costs of holding collateral form a rationale for the existence of an interbank market for reserves. Neyer (2009) demonstrates that a specific remuneration of required reserves increases the flexibility of monetary policy. Since 2008 a huge literature has evolved dealing with monetary policy implementation during the financial crisis.[13] Borio and Disyatat (2009) emphasize the importance of the central bank balance sheet size in the implemented non-standard policy measures. Cheun et al. (2009) consider changes to the collateral frameworks of the Eurosystem, the Federal Reserve System, and the Bank of England. Lenza et al. (2010) describe the different ways in which these three central banks generally conducted monetary policy during the financial crisis. Eisenschmidt et al. (2009) analyze the relatively aggressive bidding behavior of banks in the ECB's main refinancing operations at the beginning of the financial turmoil. Referring to the first part of the financial crisis (until 2008) too, Cassola and Huetl (2010) assess the effectiveness of monetary policy implementation during that time. Hauck and Neyer (2014) develop a theoretical model that considers the main institutional features of the ECB's operational framework, which has been in place since September 2008 to explain several stylized facts observed during the financial crisis. Bech and Monnet (2016) develop a search-based model of an over-the-counter (OTC) interbank market which also allows to explain several stylized facts observed in the euro area in the recent period of unconventional monetary policy measures. In their model, banks are exposed to settlement risk, which is comparable to Poole's late payment shock. This risk is responsible for the banks' use of the central bank's standing facilities. The literature emphasizing an

[12] Under- and overbidding behavior refers to a bidding behavior in which total bids significantly deviate from the Eurosystem's benchmark allotment. Analyses with respect to this under- and overbidding behavior can be found in, for example, Ayuso and Repullo (2001, 2003), Ewerhart (2002), Nautz and Oechssler (2003, 2006), and Bindseil (2005).

[13] For a description of the implementation of monetary policy by the Eurosystem in response to the financial crisis, see European Central Bank (2009, 2010, 2011, 2012b, 2014, 2015).

active role of the central bank's standing facilities in monetary policy implementation is still rather new. Pérez-Quirós and Rodríguez-Mendizábal (2006) show that the standing facilities offered by the ECB, in combination with its minimum reserve system, is an effective instrument to stabilize the interbank rate. Whitesell (2006) looks at a minimum reserve system and standing facilities as two alternative regimes for controlling overnight interest rates. Berentsen and Monnet (2008) develop a general equilibrium framework and show that changing the rates on these facilities may be actively used as a monetary policy instrument. Colliard et al. (2017) develop a core-periphery model of an OTC overnight interbank market. They argue that in a crisis a segmentation between core and periphery markets leads to a problematic dispersion of interbank rates, requiring the central bank to implement a floor operating system with respect to their interest corridor.[14] Using a theoretical model, Link and Neyer (2017) show that interbank market transaction cost heterogeneity (bank- and time-specific) leads to interbank rate volatility, and they discuss possibilities for the central bank to control the volatility using alternative interest corridor systems. Alper et al. (2013), Kara (2016), and Küçük et al. (2016) describe and discuss the active role assigned to the interest corridor in the operational framework of the Turkish Central Bank that uses the interest corridor to smooth the volatility of cross-border capital flows, i.e., as a monetary policy tool for macroprudential purposes.

Our paper combines all three described strands of literature by analyzing the impact of interbank market frictions on bank loan supply when banks face idiosyncratic liquidity risk and by discussing the resulting implications for monetary policy implementation. With respect to the latter, we emphasize the active role of the central bank's standing facilities. We argue that they not only provide an alternative to the costly interbank market for balancing liquidity positions but the facility rates also determine the costs of friction-induced holdings of positive or negative precautionary liquidity.

[14] In a floor operating system the central bank injects ample liquidity into the banking system, so that excess liquidity drives the interbank rate down to its lower bound, the rate on the deposit facility.

3 Setup

3.1 Bank Loan Supply and Banks' Liquidity Needs

Our model economy is populated by three types of agents. There is a central bank, a continuum of measure one of price-taking, risk-neutral commercial banks, and a large number of bank customers. The commercial banks make loans to their customers. The loan supply of an individual commercial bank is denoted by L_b. The interest rate a customer has to pay for a bank loan is i^L. Managing these loans generates costs $\frac{1}{2}\lambda L_b^2$ for a bank. The quadratic form of this cost function captures the idea that loans differ in their complexity so that a bank adds the least complex loans to its portfolio first.

By making loans, banks create money. They credit the respective amount to their customers' deposit accounts. Bank customers can hold this newly created money on their accounts or use it to make payments. They can pay in cash, then they have to withdraw the respective amount of cash from their accounts, or by bank transfer. The share of the new money used for cash payments is captured by the currency ratio c. Consequently, each bank faces a loss of deposits due to cash withdrawals equal to cL_b. The cash withdrawals generate a structural liquidity deficit for the whole banking sector that can only be covered by the central bank, that is the monopoly producer of currency.

In contrast, bank transfer payments do not generate a structural liquidity deficit of the banking sector as a whole, as one bank's deposit outflow is another bank's deposit inflow. However, a single bank may lose or gain deposits. The loss or gain in deposits is expressed as share χt_b of the remaining deposits $(1-c)L_b$ with χ being a scale parameter and t_b denoting the bank transfer ratio. If $t_b > 0$, the bank will face a deposit outflow due to the bank customers' transfer payments. If $t_b < 0$, there will be a deposit inflow. The bank transfer ratio differs across banks. It is the realization of the random variable $\tilde{t}_b$ which is distributed across banks according to the distribution function $G(t_b)$, with density $g(t_b)$, support $[t^{min}, t^{max}]$, and $-1 \leq t^{min} < 0 < t^{max} \leq 1$. The scale parameter χ

with $\chi \in (0, \frac{1}{t^{max}}]$ reflects the dispersion of the distribution of χt_b.[15] To sum up, a bank's liquidity needs resulting from cash withdrawals and bank transfer payments are

$$(c + (1 - c)\chi \tilde{t}_b)L_b. \tag{1}$$

A bank balances these liquidity needs by transacting with the central bank and/or in the interbank market. To obtain liquidity from the central bank, a bank can participate in the central bank's refinancing operations and borrow the amount RO_b at the policy rate i^{RO}. The central bank fully satisfies a bank's liquidity demand in these operations. A bank can also use a lending facility to borrow LF_b at the rate i^{LF}.[16] In addition, it can place an amount DF_b of liquidity in a deposit facility offered by the central bank at the rate i^{DF}. The rates on the facilities form a corridor around the policy rate i^{RO} with $i^{LF} > i^{RO} > i^{DF}$.

A bank's position in the interbank market is B_b. If $B_b > 0$, the bank will borrow the amount B_b at the rate i^{IBM}. Conversely, $B_b < 0$ indicates that the bank will lend the amount $|B_b|$ at this rate. If it borrows or lends in the interbank market, transaction costs $\gamma |B_b|$ will accrue, with $\gamma \geq 0$. These transaction costs are broadly defined as explicated in the introduction.

3.2 Sequence of Events and Optimization Problem

The sequence of events is as follows. First, each bank decides on its loan supply L_b and its borrowing from the central bank's refinancing operations RO_b. At this stage, each bank does not yet know the realization of $\tilde{t}_b$ and, therefore, its individual liquidity needs. However, it knows the distribution of $\tilde{t}_b$, so that it can form respective expectations. After each bank has decided on RO_b and L_b, bank customers withdraw cash and make their bank transfer payments. Then, each bank has to decide whether to balance its resulting liquidity position by using the interbank market and/or the central bank's facilities.

[15] As the *share* χt_b cannot exceed one, χ is restricted to $\frac{1}{t^{max}}$.

[16] Generally, credit operations with the central bank require adequate collateral. However, the collateralization of central bank credits would not qualitatively change our results (see section 6.5 for details) so that, for the sake of simplicity, we ignore this aspect in our model.

The sequence of events implies that a bank's optimization problem can be split up into two stages. At the second stage, a bank aims to minimize its liquidity costs C_b of using the interbank market and the facilities. This optimization problem reads

$$\min_{B_b \in \mathbb{R}; LF_b, DF_b \in \mathbb{R}_0^+} C_b\left(B_b, LF_b, DF_b\right) = i^{IBM} B_b + \gamma |B_b| + i^{LF} LF_b - i^{DF} DF_b \tag{2}$$

$$\text{s.t.} \qquad \left(c + (1-c)\chi t_b\right) L_b - RO_b = B_b + LF_b - DF_b. \tag{3}$$

The first two components of the cost function (2) reflect a bank's costs and possible revenues resulting from its transactions in the interbank market. The last two components represent its costs and revenues of using the central bank's facilities. Equation (3) describes a bank's balance sheet constraint. The left-hand side (LHS) reflects a bank's liquidity position at the beginning of the second stage, while the right-hand side (RHS) shows how this position is balanced.

At the first stage, each bank aims to maximize its expected profit. Denoting the respective objective function by f and indicating the optimum variables of the second stage by the superscript *opt*, the optimization problem reads

$$\max_{L_b, RO_b \in \mathbb{R}_0^+} f(L_b, RO_b) = i^L L_b - \tfrac{1}{2}\lambda L_b^2 - i^{RO} RO_b - E\left[C_b\left(B_b^{opt}, LF_b^{opt}, DF_b^{opt}\right)\right]. \tag{4}$$

The first term of the objective function shows a bank's interest revenues from making loans to the non-banking sector. The second term describes its management costs. The third term reflects the liquidity costs which accrue from borrowing from the central bank's refinancing operation. The last term represents the expected liquidity costs of using the interbank market and the central bank's facilities. Note that ex-ante, i.e., before bank customers make their payments, all banks are identical. Accordingly, all banks face the same optimization problem at the first stage and thus form the same expectations about their subsequent liquidity needs. As a deposit outflow of one bank corresponds to a respective inflow of another bank, the expected value of a bank's transfer ratio is

$$E[t_b] = \int_{t^{min}}^{t^{max}} t_b g(t_b)\, dt_b = 0. \tag{5}$$

To derive a bank's optimal behavior, we use the subgame perfect equilibrium concept and solve the optimization problem by backward induction. First, we investigate the second stage of the model determining a bank's optimal behavior on the interbank market and its optimal use of the central bank's facilities. Then, we analyze the first stage of the model and derive a bank's optimal borrowing from the central bank's refinancing operations and its optimal loan supply to the non-banking sector.

4 Optimal Behavior of a Single Bank

4.1 Optimal Behavior at the Second Stage

Considering (1) and a bank's borrowing from the central bank's refinancing operations, its liquidity position at the beginning of the second stage is

$$N_b := (c + (1-c)\chi t_b)\, L_b - RO_b \gtreqless 0. \tag{6}$$

If $N_b > 0$, the bank will face a liquidity deficit which has to be balanced by borrowing from the interbank market and/or the lending facility. Therefore, the bank compares the marginal costs of borrowing from the interbank market given by $i^{IBM} + \gamma$ with those of using the lending facility, which are i^{LF}. The bank will cover its total liquidity deficit by borrowing from the lending facility if $i^{IBM} + \gamma > i^{LF}$. If $i^{IBM} + \gamma \leq i^{LF}$, it will borrow exclusively from the interbank market. If $N_b < 0$, there will be a liquidity surplus. The excess liquidity will be placed in the interbank market or the deposit facility depending on which alternative provides the largest marginal revenues. Note that if the marginal costs (marginal revenues) of using the interbank market and of using the central bank's lending (deposit) facility are identical, the bank will obviously be indifferent between both options. However, in such a case we break ties in favor of the interbank market. If the bank's liquidity position is balanced ($N_b = 0$), the bank will use neither the interbank market nor the facilities. From (6) we can infer that $N_b = 0$ if

$$t_b = \tfrac{RO_b - cL_b}{(1-c)\chi L_b} =: \bar{t}_b. \tag{7}$$

We denote $\bar{t}_b$ as the critical bank transfer ratio. This critical ratio plays a crucial role in our analysis as it determines whether a bank faces a liquidity deficit or surplus at the beginning of the second stage.

4.2 Optimal Borrowing from the Refinancing Operations

Solving the optimization problem given by (4) for RO_b^{opt}, we obtain

Lemma 1: *Suppose that* $i^{IBM} \in \left[i^{RO} - \gamma, i^{RO} + \gamma\right]$ *and that the non-negativity constraint for* RO_b *is not binding.*[17] *Then, differentiating* f *partially with respect to* RO_b *gives the first-order condition (FOC) for a bank's optimal borrowing from the refinancing operations* RO_b^{opt}*:*

$$i^{RO} = \max\left\{i^{IBM} - \gamma, i^{DF}\right\} G\left(\bar{t}_b^{opt}\right) + \min\left\{i^{IBM} + \gamma, i^{LF}\right\}\left(1 - G\left(\bar{t}_b^{opt}\right)\right) \tag{8}$$

$$\textit{with} \quad \bar{t}_b^{opt} = \frac{RO_b^{opt} - cL_b^{opt}}{(1-c)\chi L_b^{opt}}. \tag{9}$$

Proof: See appendix.

The LHS of the FOC for a bank's optimal borrowing from the central bank's refinancing operations shows marginal costs, the RHS expected marginal revenues.[18] With probability $G(\bar{t}_b^{opt})$, a bank will face a liquidity surplus at the beginning of the second stage. In this case, any additional borrowing from the refinancing operations will generate additional interest revenues (net of possible transaction costs) as the additional surplus will be lent to the interbank market or placed in the deposit facility. With probability $1 - G(\bar{t}_b^{opt})$, a bank will face a liquidity deficit. Borrowing more from the refinancing operation at the

[17] An interbank rate $i^{IBM} \notin \left[i^{RO} - \gamma, i^{RO} + \gamma\right]$ is not a possible equilibrium rate as it would imply $RO_b^{opt} = 0$ or $RO_b^{opt} \to \infty$, respectively. At the first stage, all banks are identical. Therefore, $RO_b^{opt} = 0$ or $RO_b^{opt} \to \infty$ would imply that also at the aggregate level borrowing from the refinancing operations would be zero or infinite so that aggregate supply or demand in the interbank market at the second stage would be zero, so that there would not be an active interbank market. For the sake of a clear presentation, we thus exclude the non-relevant interbank rates already at this stage. Moreover, we do not explicitly consider the non-negativity constraint on RO_b in the presentation of our model, but we will comment on this aspect if relevant (see footnotes 22 and 29 to 35) and consider it in the formal proof of Lemma 1 given in the appendix.

[18] Note that at the first stage, a bank only forms expectations about the *individual liquidity needs* it will face at the second stage. The *interbank rate* is known for certain: All banks face exactly the same optimization problem at the first stage, so RO_b^{opt} is identical for all banks. As there is no aggregate uncertainty, each bank therefore knows the aggregate liquidity endowment of the banking sector and thus the interbank rate for any RO_b^{opt}.

first stage will reduce the liquidity deficit at the second stage, so that in this case marginal revenues in the form of avoided illiquidity costs will accrue.

In our analysis, the interbank rate that balances the marginal costs and the expected marginal revenues of borrowing from the refinancing operations for $RO_b = cL_b$ plays an important role. We denote this rate by $\tilde{i}^{IBM}$. Considering (8), we obtain

$$\tilde{i}^{IBM} = i^{RO} - \gamma(1 - 2G(0)). \tag{10}$$

Precautionary Liquidity Holdings

Considering (1) and (5), a bank's expected liquidity needs are cL_b. If a bank borrows more (less) than cL_b from the refinancing operations, so that $\bar{t}_b > 0$ ($\bar{t}_b < 0$), it will thus hold positive (negative) precautionary liquidity. This leads us to

Lemma 2: *A bank will hold positive (negative) precautionary liquidity if it expects for $RO_b = cL_b$, i.e., for $\bar{t}_b = 0$, that a deficit will be more (less) expensive at the margin than a surplus, i.e., if*

$$\left(i^{RO} - \max\left\{i^{IBM} - \gamma, i^{DF}\right\}\right) G(0) \gtrless \left(\min\left\{i^{IBM} + \gamma, i^{LF}\right\} - i^{RO}\right)(1 - G(0)). \tag{11}$$

The LHS of (11) presents the expected marginal costs of a surplus, the LHS of a deficit. From (11) we can conclude that if a bank holds precautionary liquidity, some kind of asymmetry will be involved. In our model, there are two possible sources of asymmetry: an asymmetric distribution of $\tilde{t}_b$ around zero ($G(0) \neq 0$) and an asymmetric interest corridor ($i^{LF} - i^{RO} \neq i^{RO} - i^{DF}$). However, one can think of other possible sources, such as stigma costs when borrowing from the interbank market or different transaction costs for interbank market lending and borrowing. Rearranging (8), we obtain the FOC for optimal precautionary liquidity holdings:

$$\begin{aligned}&\left(i^{RO} - \max\left\{i^{IBM} - \gamma, i^{DF}\right\}\right) G\left(\bar{t}_b^{opt}\right)\\ &= \left(\min\left\{i^{IBM} + \gamma, i^{LF}\right\} - i^{RO}\right)\left(1 - G\left(\bar{t}_b^{opt}\right)\right).\end{aligned} \tag{12}$$

If a bank holds *positive* precautionary liquidity, the LHS of (12) will represent its expected marginal costs of holding this liquidity in the form of additional interest costs, while the RHS will show its expected marginal revenues in the form of avoided illiquidity costs. If a bank holds *negative* precautionary liquidity, the LHS of (12) will represent the expected marginal revenues of holding this liquidity in the form of avoided interest costs while the LHS will show respective expected marginal costs in the form of additional illiquidity costs.

4.3 Optimal Bank Loan Supply

Solving the optimization problem given by (4) for L_b^{opt}, we obtain

Lemma 3: *Differentiating f partially with respect to L_b and considering* (8)*, the FOC for a bank's optimal loan supply to the non-banking sector becomes*

$$i^L = \lambda L_b^{opt} + ci^{RO} + \phi(\bar{t}_b^{opt}) \left[\min\left\{i^{IBM} + \gamma, i^{LF}\right\} - \max\left\{i^{IBM} - \gamma, i^{DF}\right\}\right], \tag{13}$$

with $\bar{t}_b^{opt}$ defined by (9) *and*

$$\phi(\bar{t}_b^{opt}) := (1-c)\chi \int_{\bar{t}_b^{opt}}^{t^{max}} t_b g(t_b)\, dt_b. \tag{14}$$

Proof: See appendix.

The LHS of the FOC for a bank's optimal loan supply presents the marginal revenues of granting loans, the RHS expected marginal costs. The latter consist of the marginal management costs λL_b^{opt} and expected marginal liquidity costs. In the following, we will comment on the latter in more detail.

If a bank increases its loans by one unit, it will face additional *certain* liquidity needs c because of additional cash withdrawals. These certain liquidity needs imply certain marginal liquidity costs ci^{RO}. Furthermore, the increase in loans implies additional *uncertain* liquidity needs due to uncertain bank transfer payments $(1-c)\chi\tilde{t}_b$. And although the respective expected additional liquidity needs are zero as $E[t_b] = 0$, the bank's expected additional (net) liquidity costs will be positive if the possible deficit costs are higher than the possible surplus revenues (in this case, the term in square brackets in (13) will be positive). We refer to these costs as the uncertainty costs of granting loans. Note that if

interbank trade is not impaired ($\gamma = 0$), uncertainty costs of granting loans will be zero. The importance of these costs for a bank's optimal bank loan supply is reflected by the quantity and probability weight $\phi(\bar{t}_b^{opt})$.[19] Consequently, the last term in the FOC (13) presents *the expected marginal uncertainty costs of granting loans.* In our analysis, it plays a crucial role that the weight $\phi(\bar{t}_b^{opt})$, and thus the expected marginal uncertainty costs of granting loans, decrease in a bank's holdings of positive as well as negative precautionary liquidity:

$$\frac{\partial \phi(\bar{t}_b^{opt})}{\partial \bar{t}_b^{opt}} = -(1-c)\chi \bar{t}_b^{opt} g(\bar{t}_b^{opt}) \begin{cases} < 0 & \text{if} \quad \bar{t}_b^{opt} > 0, \\ > 0 & \text{if} \quad \bar{t}_b^{opt} < 0. \end{cases} \tag{15}$$

5 Aggregate Equilibrium

5.1 Interbank Rate

After having clarified the optimal behavior of an individual bank, we are now in a position to look at the aggregate equilibrium. At the second stage, each bank will only trade liquidity in the interbank market if this is more beneficial than using the respective facility. All variables and parameters determining this decision are the same for all banks. Consequently, if $i^{IBM} - \gamma < i^{DF}$, all banks with a liquidity surplus will place their excess liquidity in the deposit facility, while for $i^{IBM} + \gamma > i^{LF}$, all banks with a liquidity deficit will borrow the missing liquidity from the lending facility. There is thus an upper and a lower bound for the interbank rate given by

$$\overline{i^{IBM}} = i^{LF} - \gamma, \tag{16}$$

$$\underline{i^{IBM}} = i^{DF} + \gamma. \tag{17}$$

[19] The weight of the deficit costs is $(1-c)\chi \int_{\bar{t}_b^{opt}}^{t^{max}} t_b g(t_b)\, dt_b$, the weight of the surplus revenues is $-(1-c)\chi \int_{t^{min}}^{\bar{t}_b^{opt}} t_b g(t_b)\, dt_b$. Note that $\int_{\bar{t}_b^{opt}}^{t^{max}} t_b g(t_b)\, dt_b = E[t_b | t_b > \bar{t}_b^{opt}](1 - G(t_b))$ and that $-\int_{t^{min}}^{\bar{t}_b^{opt}} t_b g(t_b)\, dt_b = -E[t_b | t_b < \bar{t}_b^{opt}]G(t_b)$, so that the weights reflect the expected additional deficit/surplus and the respective occurrence probability. As $E[t_b] = 0$, we have $-(1-c)\chi \int_{t^{min}}^{\bar{t}_b^{opt}} t_b g(t_b)\, dt_b = (1-c)\chi \int_{\bar{t}_b^{opt}}^{t^{max}} t_b g(t_b)\, dt_b$,, which means that the weight with which possible deficit costs and surplus revenues enter the FOC is the same. Capturing this identical weight by $\phi(\bar{t}_b^{opt})$, the expected marginal uncertainty costs of granting loans simplify to the last term in (13).

We denote aggregate borrowing from the refinancing operations by RO and aggregate loan supply to the non-banking sector by L. An aggregate liquidity deficit will arise if the banking sector's structural liquidity needs cL exceed the aggregate amount borrowed from the refinancing operations RO. In this case, competition for scarce liquidity will bring the interbank rate to its upper bound $\overline{i^{IBM}}$. If an aggregate liquidity surplus occurs, competition for limited lending possibilities in the interbank market will bring the interbank rate to its lower bound $\underline{i^{IBM}}$. If there is neither an aggregate liquidity deficit nor surplus ($RO = cL$), any rate within the interval

$$I := [\underline{i^{IBM}}, \overline{i^{IBM}}] \tag{18}$$

can be consistent with an equilibrium. The upper and the lower bound for i^{IBM} lead us to an upper threshold for γ. If γ exceeds

$$\bar{\bar{\gamma}} := \frac{i^{LF} - i^{DF}}{2}, \tag{19}$$

the interbank market will break down. Transaction costs are prohibitively high. Both, the upper and the lower bound for the interbank rate will become binding, i.e., $I|_{\gamma > \bar{\bar{\gamma}}} = \{\}$. Denoting (subgame perfect) equilibrium variables by an asterisk, these considerations lead us to

Proposition 1: *If $\gamma \leq \bar{\bar{\gamma}}$, there will be an active interbank market at the second stage, and the subgame perfect equilibrium interest rate i^{IBM*} at the first stage will be*

$$i^{IBM*} = \begin{cases} \underline{i^{IBM}} & \text{if} \quad RO^* > cL^*, \\ \tilde{i}^{IBM} \in I & \text{if} \quad RO^* = cL^*, \\ \overline{i^{IBM}} & \text{if} \quad RO^* < cL^*, \end{cases} \tag{20}$$

where $\tilde{i}^{IBM}$ is defined by (10).

At the second stage of the model, any $i^{IBM} \in I$ depicts an equilibrium for $RO^* = cL^*$. However, at the first stage, only $\tilde{i}^{IBM}$ as given by (10) will be the subgame perfect equilibrium interest rate for $RO^* = cL^*$. For all other $i^{IBM} \in I$, the expected marginal revenues of borrowing from the refinancing operations strictly deviate from marginal costs, inducing

banks to borrow more or less than cL from the refinancing operations. Proposition 1 leads us to a further threshold for γ:[20]

$$\bar{\gamma} := \min\left\{\bar{\gamma}_l := \frac{i^{RO} - i^{DF}}{2(1-G(0))}, \bar{\gamma}_u := \frac{i^{LF} - i^{RO}}{2G(0)}\right\} \leq \bar{\bar{\gamma}}. \tag{21}$$

If $\gamma \leq \bar{\gamma}$, we will have $\tilde{i}^{IBM} \in I$, i.e., borrowing $RO = cL$ will balance the banks' expected marginal revenues and the marginal costs of borrowing from the refinancing operations. If $\bar{\gamma} < \gamma \leq \bar{\bar{\gamma}}$, this will no longer be the case, $\tilde{i}^{IBM} \notin I$. If $\bar{\gamma} = \bar{\gamma}_l < \gamma \leq \bar{\bar{\gamma}}$, we will have $\tilde{i}^{IBM} < \underline{i^{IBM}}$, and if $\bar{\gamma} = \bar{\gamma}_u < \gamma \leq \bar{\bar{\gamma}}$, we will have $\tilde{i}^{IBM} > \overline{i^{IBM}}$. This means that if $\bar{\gamma} < \gamma \leq \bar{\bar{\gamma}}$, there will be an active interbank market, but for all $i^{IBM} \in I$ expected marginal revenues of borrowing from the refinancing operations will deviate from marginal costs for $RO = cL$. Banks will thus borrow more or less than cL from the refinancing operations as we will show in the next section.

5.2 Borrowing from the Refinancing Operations and Lending to the Non-Banking Sector

At the first stage, all banks face the same optimization problem, so that RO_b^{opt}, L_b^{opt} and, therefore, also $\bar{t}_b^{opt}$ are identical for all banks. As we have a continuum of banks of unit mass, these bank-individual optimal values correspond to the respective aggregate equilibrium variables L^* and RO^*. Furthermore, we can use $\bar{t}^{opt} = \bar{t}^*$ as the critical equilibrium bank transfer ratio that is identical for all banks. Depending on the level of interbank market transaction costs, three different equilibria indexed by $j = I, II, III$ are possible. The main characteristics of these equilibria are:

- Equilibrium I: Low transaction costs, active interbank market, no precautionary liquidity holdings.
- Equilibrium II: High transaction costs, active interbank market, precautionary liquidity holdings.
- Equilibrium III: Prohibitively high transaction costs, inactive interbank market, possible precautionary liquidity holdings.

[20] One obtains $\bar{\gamma}_l$ by setting in (10) $\tilde{i}^{IBM} = \underline{i^{IBM}}$ and solving that equation for γ. Analogously, one obtains $\bar{\gamma}_u$ by setting in (10) $\tilde{i}^{IBM} = \overline{i^{IBM}}$.

In the following, we will have a closer look at each of them.

5.2.1 Equilibrium I: Low Transaction Costs

Proposition 2: *If $\gamma \leq \bar{\gamma}$ banks will exclusively use the interbank market to balance their liquidity position at the second stage, i.e.,*

$$RO^{*I} = cL^{*I}, \quad \textit{so that} \quad \bar{t}^{*I} = \frac{RO^{*I} - cL^{*I}}{(1-c)\chi L^{*I}} = 0 \quad \textit{and} \quad i^{IBM*I} = \tilde{i}^{IBM} \in I.$$

Borrowing from the Refinancing Operations

If interbank market transaction costs are sufficiently low ($\gamma \leq \bar{\gamma}$), borrowing $RO = cL$ will balance banks' expected marginal revenues and the marginal costs of borrowing from the refinancing operations. Borrowing more (less) than cL from the refinancing operations would result in an aggregate liquidity surplus (deficit) at the beginning of the second stage bringing the interbank rate to its lower (upper) bound. This low (high) interbank rate would imply that the expected marginal revenues of borrowing from the refinancing operations would strictly deviate from marginal costs.[21] Consequently, if $\gamma \leq \bar{\gamma}$, $RO^{*I} = cL^{*I}$ will be the only feasible equilibrium.

Note that for this result a possible asymmetry of the interest corridor and/or of the distribution of $\tilde{t}_b$ does not play a role. The rates on the facilities are irrelevant for the banks' borrowing decision, as with $\gamma \leq \bar{\gamma}$ transaction costs are so low that no bank expects to use the central bank's facilities. With respect to a possible asymmetric distribution of $\tilde{t}_b$ consider that rising transaction costs increase the costs of both, balancing a surplus and a deficit. However, an asymmetric distribution of $\tilde{t}_b$ ($G(0) \neq 0.5$) means that for $RO = cL$ banks expect a deficit to be more or less probable so that rising transaction costs will increase the expected costs of a deficit more or less than those of a surplus. This should induce banks to hold positive or negative precautionary liquidity. However, as long as $\gamma \leq \bar{\gamma}$, there will be a change in $i^{IBM*I} = \tilde{i}^{IBM}$ exactly compensating the effect of an increase in γ on marginal surplus and deficit costs, i.e., the price mechanism

[21] With an active interbank market, the FOC for optimal borrowing from the refinancing operations given in Lemma 1 becomes $(i^{IBM*} - \gamma)G\left(\bar{t}\right) + (i^{IBM*} + \gamma)\left(1 - G\left(\bar{t}\right)\right) = i^{RO}$. When borrowing $RO = cL$, $i^{IBM*} = \tilde{i}^{IBM}$ and $G(\bar{t}) = G(0)$ which implies that the FOC is fulfilled. When borrowing more than cL, $i^{IBM*} = \underline{i^{IBM}} = i^{DF} + \gamma$ and $G(\bar{t}) > G(0)$ which implies that expected marginal revenues of borrowing from the refinancing operations are strictly lower than marginal costs. When borrowing less than cL, $i^{IBM*} = \overline{i^{IBM}} = i^{LF} - \gamma$ and $G(\bar{t}) < G(O)$ implying that the expected marginal revenues are strictly higher than marginal costs.

in the interbank market functions, so that $RO^{*I} = cL^{*I}$ remains the equilibrium solution. Formally, this can be seen by having a closer look at Lemma 1. In case of an active interbank market, (8) can be rewritten as

$$(i^{RO} - i^{IBM*j} + \gamma)G(\bar{t}^{*j}) = (i^{IBM*j} + \gamma - i^{RO})\left(1 - G(\bar{t}^{*j})\right), \quad \text{with } j = I, II. \tag{22}$$

The LHS of (22) shows the expected marginal costs of a surplus, the RHS of a deficit. In Equilibrium I, $i^{IBM*I} = \tilde{i}^{IBM}$ as given by (10) and $\bar{t}^{*I} = 0$. An increase in γ has no effect on either side, as it is exactly compensated by a respective decrease in i^{IBM*I}.

Lending to the Non-Banking Sector

Lemma 3 reveals that interbank market transaction costs are a crucial determinant of banks' expected marginal uncertainty costs of granting loans. For $\gamma \leq \bar{\gamma}$, we have $RO^{*I} = cL^{*I}$, i.e. $\bar{t}^{*I} = 0$, and all banks prefer to use the interbank market instead of the facilities. Expected marginal uncertainty costs of granting loans thus simplify to

$$\phi(0)2\gamma, \quad \text{where} \quad \phi(0) = (1-c)\chi \int_0^{t^{max}} t_b g(t_b)\, dt_b. \tag{23}$$

If interbank trading is not impaired ($\gamma = 0$), expected marginal uncertainty costs will be zero. Expected deficit costs and expected surplus revenues will be the same, both will equal the interbank rate. However, positive transaction costs mean higher costs when borrowing from the interbank market and lower revenues when lending to this market, i.e., they put a wedge between the costs and revenues of trading in the interbank market, leading to positive expected marginal uncertainty costs.

5.2.2 Equilibrium II : High Transaction Costs

Proposition 3: *If $\bar{\gamma} = \bar{\gamma}_l < \gamma \leq \bar{\bar{\gamma}}$, banks will hold positive precautionary liquidity and will use the interbank market and the deposit facility to balance their liquidity position at the second stage, i.e.,*

$$RO^{*II} > cL^{*II}, \quad \textit{so that} \quad \bar{t}^{*II} = \frac{RO^{*II} - cL^{*II}}{(1-c)\chi L^{*II}} > 0 \quad \textit{and} \quad i^{IBM*II} = \underline{i^{IBM}}.$$

If $\bar{\gamma} = \bar{\gamma}_u < \gamma \leq \bar{\bar{\gamma}}$, banks will hold negative precautionary liquidity and will use the interbank market and the lending facility to balance their liquidity position at the second stage, i.e.,[22]

$$RO^{*II} < cL^{*II}, \quad \textit{so that} \quad \bar{t}^{*II} = \frac{RO^{*II} - cL^{*II}}{(1-c)\chi L^{*II}} < 0 \quad \textit{and} \quad i^{IBM*II} = \overline{i^{IBM}}.$$

Borrowing from the Refinancing Operations

If interbank market transaction costs are sufficiently high ($\gamma > \bar{\gamma}$), possible asymmetries of the interest corridor and the distribution of $\tilde{t}_b$ will be relevant for banks' optimal borrowing from the refinancing operations. Due to the asymmetries, banks expect a deficit to be more or less expensive than a surplus for $RO = cL$, so that they hold precautionary liquidity.

If we have a *symmetric distribution* of $\tilde{t}_b$ ($G(0) = 0.5$) but an *asymmetric interest corridor* in the form of $i^{LF} - i^{RO} > i^{RO} - i^{DF}$, an increase in γ beyond $\bar{\gamma} = \bar{\gamma}_l$ will imply that at first only banks with a liquidity surplus will no longer be willing to use the interbank market but will prefer the central bank's deposit facility, whereas for banks with a liquidity deficit, it will be still more beneficial to use the interbank market. To maintain an active interbank market, deficit banks thus accept having to also bear the transaction costs of the surplus banks.[23] Note that if γ exceeds $\bar{\bar{\gamma}}$, transaction costs will be so high, that deficit banks will no longer be willing to bear double transaction costs, but will prefer to use the central bank's facilities. The interbank market will break down (Equilibrium III). However, as long as we have an active interbank market, due to the higher transaction cost burden, banks expect a deficit to be more expensive than a surplus inducing them to hold positive precautionary liquidity, $RO^{*II} > cL^{*II}$.[24] Analogously, banks will hold negative precautionary liquidity ($RO^{*II} < cL^{*II}$) if $i^{LF} - i^{RO} < i^{RO} - i^{DF}$ and if $\bar{\gamma} = \bar{\gamma}_u < \gamma \leq \bar{\bar{\gamma}}$.

Let us turn next to the case of a *symmetric interest corridor* and an *asymmetric distribution* of $\tilde{t}_b$ in the form of $G(0) < 0.5$. The asymmetric distribution means that for $RO = cL$ banks expect a deficit to be more probable than a surplus. As long as

[22] Note that if banks hold negative precautionary liquidity and if the non-negativity constraint on RO_b binds, $RO^{*II} = 0$ and $\bar{t}^{*II} = \underline{t} = -c/(1-c)\chi < 0$ (see proof of Lemma 1 in the appendix).

[23] In this case, balancing a liquidity deficit in the interbank market costs $i^{IBM*II} + \gamma = \underline{i^{IBM}} + \gamma = i^{DF} + 2\gamma$).

[24] If $\bar{\gamma} = \bar{\gamma}_l < \gamma \leq \bar{\bar{\gamma}}$, $\tilde{i}^{IBM} < \underline{i^{IBM}}$ but $I \neq \{\}$. This means that if banks borrowed $RO = cL$, for all $i^{IBM} \in I$ the expected marginal revenues of borrowing from the refinancing operations would be higher than the expected marginal costs. Consequently, $RO^{*II} > cL^{*II}$. Banks start to hold positive precautionary liquidity until the FOC for optimal borrowing from the refinancing operations with an active interbank market (22) with $i^{IBM*II} = \underline{i^{IBM}}$ and $\bar{t}^{*II} > 0$ is fulfilled.

$\gamma \leq \bar{\gamma} = \bar{\gamma}_l$, the resulting stronger impact of an increase in γ on expected marginal deficit costs rather than on surplus costs will be compensated by a respective decrease in $\tilde{i}^{IBM}$ (Equilibrium I). However, if γ reaches $\bar{\gamma}_l$, further decreases in $\tilde{i}^{IBM}$ will no longer be possible, the price mechanism in the interbank market will not function any longer as the lower bound for i^{IBM} will become binding. Then, further increases in γ mean that banks expect a deficit to be more expensive than a surplus so that they start to hold positive precautionary liquidity, $RO^{*II} > cL^{*II}$.[25] A similar story can be told for $G(0) > 0.5$. Then, $\bar{\gamma} = \bar{\gamma}_u$, and if $\bar{\gamma} = \bar{\gamma}_u < \gamma \leq \bar{\bar{\gamma}}$, banks will start to hold negative precautionary liquidity, $RO^{*II} < cL^{*II}$.

If the *distribution* of $\tilde{t}_b$ as well as the *interest corridor are asymmetric*, and if both asymmetries work in the same (opposite) direction, given the extent of one asymmetry, the additional asymmetry leads to an even lower (higher) $\bar{\gamma}$, i.e., banks start to hold precautionary liquidity for even lower (higher) interbank market transaction costs.

In our discussion on monetary policy implementation in Section 6, the FOCs for optimal holdings of precautionary liquidity play an important role. Considering (12) and i^{IBM*II} being equal to $\underline{i^{IBM}}$, the FOC for optimal holdings of positive precautionary liquidity becomes

$$(i^{RO} - i^{DF})G(\bar{t}^{*II}) = (i^{DF} + 2\gamma - i^{RO})(1 - G(\bar{t}^{*II})), \quad \text{with } \bar{t}^{*II} > 0. \tag{24}$$

The expected marginal costs of holding positive precautionary liquidity in the form of interest costs are given by the LHS of (24). The RHS shows the expected marginal revenues in the form of avoided illiquidity costs.[26] Analogously, the FOC for holding optimal negative precautionary liquidity is given by

$$(i^{RO} - i^{LF} + 2\gamma)G(\bar{t}^{*II}) = (i^{LF} - i^{RO})(1 - G(\bar{t}^{*II})), \quad \text{with } \bar{t}^{*II} < 0. \tag{25}$$

The expected marginal costs of holding negative precautionary liquidity in the form of higher interest costs in case a deficit occurs, are presented by the RHS of (25). The LHS

[25] Formally, the same arguments as made in footnote 24 hold.

[26] Holding positive precautionary liquidity allows banks to finance a possible deficit at the rate of i^{RO} instead of $i^{IBM*II} + \gamma = i^{DF} + 2\gamma$ which is strictly larger than i^{RO} for $\gamma > \bar{\gamma}_l$.

shows expected marginal revenues in the form of avoided surplus costs.[27]

Lending to the Non-Banking Sector

It follows from Lemma 3, that for $\bar{\gamma} < \gamma \leq \bar{\bar{\gamma}}$ the expected marginal uncertainty costs of granting loans become

$$\phi(\bar{t}^{*II})2\gamma \quad \text{where} \quad \phi(\bar{t}^{*}) = (1-c)\chi \int_{\bar{t}^{*II}}^{t^{max}} t_b g(t_b)\, dt_b. \tag{26}$$

Again, positive transaction costs put a wedge between the costs and revenues of using the interbank market.[28] Expected marginal uncertainty costs of granting loans are thus positive and increase in γ. However, the more precautionary liquidity banks hold, the lower the costs will be (see Section 4.3 for details).

5.2.3 Equilibrium III: Prohibitively High Transaction Costs

Proposition 4: *If $\gamma > \bar{\bar{\gamma}}$, the interbank market will break down and banks only use the central bank's facilities to balance their liquidity position at the second stage. Depending on the relative asymmetries of the distribution of $\tilde{t}_b$ and of the interest corridor, banks may hold positive, negative or no precautionary liquidity:*[29]

$$RO^{*III} \begin{cases} > cL^{*III} & \Rightarrow \bar{t}^{*III} = \frac{RO^{*III} - cL^{*III}}{(1-c)\chi L^{*III}} > 0 \quad \textit{if} \quad G(0) < \frac{i^{LF} - i^{RO}}{i^{LF} - i^{DF}}, \\ = cL^{*III} & \Rightarrow \bar{t}^{*III} = \frac{RO^{*III} - cL^{*III}}{(1-c)\chi L^{*III}} = 0 \quad \textit{if} \quad G(0) = \frac{i^{LF} - i^{RO}}{i^{LF} - i^{DF}}, \\ < cL^{*III} & \Rightarrow \bar{t}^{*III} = \frac{RO^{*III} - cL^{*III}}{(1-c)\chi L^{*III}} < 0 \quad \textit{if} \quad G(0) > \frac{i^{LF} - i^{RO}}{i^{LF} - i^{DF}}. \end{cases}$$

Borrowing from the Refinancing Operations

If $\gamma > \bar{\bar{\gamma}}$, interbank market transaction costs will be prohibitively high, so that all banks will exclusively use the central bank's facilities to balance their liquidity position at the second stage. Whether they hold precautionary liquidity depends again on the asymme-

[27] Surplus costs accrue because banks facing a liquidity surplus at the beginning of the second stage, have borrowed this liquidity at the rate of i^{RO} and receive $i^{IBM*II} - \gamma = i^{LF} - 2\gamma < i^{RO}$ for $\gamma > \bar{\gamma}_u$ when lending it to the interbank market.

[28] If $\bar{t}^{*II} > 0$, the interbank rate will be at its lower bound so that the marginal costs of balancing a deficit are $i^{DF} + 2\gamma$, whereas the marginal revenues of balancing a surplus are i^{DF}. If $\bar{t}^{*II} < 0$, the interbank rate will be at its upper bound so that the marginal costs of balancing a deficit are i^{LF}, whereas marginal revenues of balancing a surplus are $i^{LF} - 2\gamma$.

[29] Note again that if banks hold negative precautionary liquidity and if the negativity-constraint on RO_b binds, $RO^{*III} = 0$ and $\bar{t}^{*III} = \underline{t} = -c/(1-c)\chi < 0$ (see proof of Lemma 1 in the appendix).

tries of the distribution of $\tilde{t}_b$ and of the interest corridor. According to (11), for $RO = cL$ the expected marginal costs of a deficit are $(1 - G(0))(i^{LF} - i^{RO})$, of a surplus these costs are $G(0)(i^{RO} - i^{DF})$. Consequently, an asymmetric distribution ($G(0) \neq 0.5$) and/or an asymmetric interest corridor ($i^{LF} - i^{RO} \neq i^{RO} - i^{DF}$) result in a deficit or a surplus that is more expensive, so the banks hold precautionary liquidity unless the effects of the two asymmetries cancel each other out.

Referring to (12) the FOC for holding precautionary liquidity in this equilibrium reads

$$\left(i^{RO} - i^{DF}\right) G\left(\bar{t}^{*III}\right) = \left(i^{LF} - i^{RO}\right)\left(1 - G\left(\bar{t}^{*III}\right)\right) \quad \text{with} \quad \bar{t}^{*III} \gtreqless 0. \tag{27}$$

If banks hold positive precautionary liquidity ($\bar{t}^{*III} > 0$), the LHS of (27) presents the expected marginal costs of holding this liquidity in the form of interest costs. The RHS shows the expected marginal revenues in the form of avoided illiquidity costs. If banks hold negative precautionary liquidity ($\bar{t}^{*III} < 0$), the LHS will present the expected marginal revenues (avoided surplus costs), whereas the RHS will show the expected marginal costs (interest costs).

Lending to the Non-Banking Sector

Referring to Lemma 3, if the interbank market breaks down, the expected marginal uncertainty costs of granting loans will be given by

$$\phi(\bar{t}^{*III})(i^{LF} - i^{DF}), \quad \text{where} \quad \phi(\bar{t}^{*III}) = (1 - c)\chi \int_{\bar{t}^{*III}}^{t^{max}} t_b g(t_b)\, dt_b. \tag{28}$$

These costs increase in the width of the interest corridor, and this increase will be less pronounced the more precautionary liquidity banks hold.

6 Monetary Policy and Bank Loan Supply

With respect to the influence of monetary policy on bank loan supply, the banks' expected marginal liquidity costs of granting loans play a crucial role. In the following, we will first analyze how interbank market transaction costs and banks' uncertainty about their idiosyncratic liquidity needs affect bank loan supply. Then, we will examine the

impact of different monetary policy impulses on bank loan supply, explicitly pointing to the importance of interbank market transaction costs and of banks' uncertainty over their idiosyncratic liquidity needs.

6.1 Frictions and Uncertainty

Result 1: *In the three possible equilibria I, II, and III, the effect of a change in interbank market transaction costs on bank loan supply is*

$$\frac{\partial L^{*I}}{\partial \gamma} = \frac{-2\phi(0)}{\lambda} < 0, \quad \frac{\partial L^{*II}}{\partial \gamma} = \left\{ \begin{array}{ll} -\frac{2\phi(\bar{t}^{*II})}{\lambda} + \frac{2(1-c)\chi \bar{t}^{*II}(1-G(\bar{t}^{*II}))}{\lambda} & \textit{if} \quad \bar{t}^{*II} > 0 \\ -\frac{2\phi(\bar{t}^{*II})}{\lambda} - \frac{2(1-c)\chi \bar{t}^{*II} G(\bar{t}^{*II})}{\lambda} & \textit{if} \quad \bar{t}^{*II} < 0 \end{array} \right\} < 0,$$

$$\frac{\partial L^{*III}}{\partial \gamma} = 0.$$

Proof: See appendix.

Result 1 shows that in Equilibria I and II, increasing interbank market transaction costs have a contractionary effect on bank loan supply. In both equilibria, banks expect to use the interbank market to balance uncertain liquidity needs, so that higher transaction costs increase the expected marginal uncertainty costs of granting loans. Holdings of precautionary liquidity reduce the expected marginal uncertainty costs (see (15)). As higher interbank market transaction costs induce banks to hold more precautionary liquidity (in absolute terms),[30] in Equilibrium II, the contractionary effect is weakened, as is formally shown by the second term of $\partial L^{*II}/\partial \gamma$.[31] In Equilibrium III, the interbank market is not (expected to be) used, so that γ has no influence on bank loan supply.

Result 2: *In the three possible equilibria I, II, and III, the effect of a change in the extent of uncertainty on bank loan supply is*

$$\frac{\partial L^{*I}}{\partial \chi} = \frac{-\phi(0)2\gamma}{\lambda\chi} < 0, \quad \frac{\partial L^{*II}}{\partial \chi} = \frac{-\phi(\bar{t}^{*II})2\gamma}{\lambda\chi} < 0, \quad \frac{\partial L^{*III}}{\partial \chi} = \frac{-\phi(\bar{t}^{*III})\left(i^{LF} - i^{DF}\right)}{\lambda\chi} < 0.$$

Proof: See appendix.

[30] Formally, this is revealed by the FOCs (24) and (25).

[31] If the non-negativity constraint on RO_b binds, banks cannot increase their negative precautionary liquidity holdings. In this case, the described weakening effect in Equilibrium II is omitted. Furthermore, $\phi(\bar{t}^{*II}) = \phi(\underline{t})$ with $\underline{t} = -c/(1-c)\chi < 0$ (see proof of Lemma 1 in the appendix).

Result 2 reveals that in all equilibria, an increase in the extent of uncertainty leads to a decrease in bank loan supply. Higher uncertainty, here in the form of a higher dispersion of $\chi\tilde{t}_b$, means that banks expect a larger deficit as well as a larger surplus. Consequently, the expected marginal uncertainty costs increase and banks reduce their loan supply. In Equilibria II and III, this effect is mitigated as the increase in χ implies that banks hold more precautionary liquidity, which has a negative effect on expected marginal uncertainty costs (see (15)).[32]

6.2 Equal Change in All Central Bank Interest Rates

Starting our analysis of the impact of different monetary policy impulses on bank loan supply with an equal change in all central bank's interest rates, we get

Result 3: *If* $\frac{di^{DF}}{di^{RO}} = \frac{di^{LF}}{di^{RO}} = 1$, *we obtain*

$$\frac{\partial L^{*j}}{\partial i^{RO}} = -\frac{c}{\lambda} < 0 \quad \textit{and} \quad \frac{\partial^2 L^{*j}}{\partial i^{RO}\partial\chi} = \frac{\partial^2 L^{*j}}{\partial i^{RO}\partial\gamma} = 0 \quad \forall \quad j.$$

Proof: See appendix.

An equal decrease in all central bank interest rates has the same expansionary effect on bank loan supply in all equilibria $j = I, II, III$. The crucial point is that this monetary policy impulse only leads to a change in certain marginal liquidity costs, whereas the expected marginal uncertainty costs of granting loans remain unchanged. It follows from (23), (26), and (28) that this monetary policy impulse would only have an impact on these costs if banks changed their holdings of precautionary liquidity, i.e., if $\bar{t}^*$ changed. However, an equal change in all interest rates has no impact on the marginal costs and revenues of holding precautionary liquidity, as revealed by the FOCs (24), (25), and (27). As γ and χ are only relevant for expected marginal uncertainty costs of granting loans, neither the interbank market friction in the form of transaction costs nor the uncertainty

[32] If the non-negativity constraint on RO_b binds, banks will not be able to increase their holdings of negative precautionary liquidity. The effect of an increase in χ on L^{*II} and L^{*III} will thus be stronger. Formally, we have

$$\frac{\partial L^{*II}}{\partial\chi} = \frac{-\phi(\underline{t})2\gamma}{\lambda\chi} + \frac{2\gamma\underline{t}g(\underline{t})c}{\lambda\chi} < 0, \quad \frac{\partial L^{*III}}{\partial\chi} = \frac{-\phi(\underline{t})\left(i^{LF} - i^{DF}\right)}{\lambda\chi} + \frac{\left(i^{LF} - i^{DF}\right)\underline{t}g(\underline{t})c}{\lambda\chi} < 0,$$

with $\underline{t} = -c/(1-c)\chi < 0$ (see proof of Lemma 1 in the appendix).

about idiosyncratic liquidity needs has an impact on the effectiveness of this monetary policy impulse on bank loan supply.

6.3 Change in the Width of the Interest Corridor

Alternatively to an equal change in all central bank interest rates, the central bank may change the width of the interest corridor around its policy rate. This leads us to

Result 4: *If* $\frac{di^{DF}}{di^{LF}} = -1$, *we obtain*

$$\frac{\partial L^{*I}}{\partial i^{LF}} = 0, \qquad \frac{\partial L^{*II}}{\partial i^{LF}} = -\frac{(1-c)\chi|\bar{t}^{*II}|}{\lambda} < 0, \qquad \frac{\partial L^{*III}}{\partial i^{LF}} = -\frac{2\phi(\bar{t}^{*III})}{\lambda} < 0,$$

$$\frac{\partial^2 L^{*I}}{\partial i^{LF}\partial\chi} = 0, \qquad \frac{\partial^2 L^{*II}}{\partial i^{LF}\partial\chi} = -\frac{(1-c)|\bar{t}^{*II}|}{\lambda} < 0, \qquad \frac{\partial^2 L^{*III}}{\partial i^{LF}\partial\chi} = -\frac{2\phi(\bar{t}^{*III})}{\lambda\chi} < 0,$$

$$\frac{\partial^2 L^{*I}}{\partial i^{LF}\partial\gamma} = 0, \qquad \frac{\partial^2 L^{*II}}{\partial i^{LF}\partial\gamma} = \left\{ \begin{array}{ll} -\frac{(1-c)\chi(1-G(\bar{t}^{*II}))}{\lambda\gamma g(\bar{t}^{*II})} & \textit{if} \quad \bar{t}^{*II} > 0 \\ -\frac{(1-c)\chi G(\bar{t}^{*II})}{\lambda\gamma g(\bar{t}^{*II})} & \textit{if} \quad \bar{t}^{*II} < 0 \end{array} \right\} < 0, \qquad \frac{\partial^2 L^{*III}}{\partial i^{LF}\partial\gamma} = 0.$$

Proof: See appendix.

In our model, the symmetry/asymmetry of the corridor refers to the refinancing rate i^{RO}. The corridor will be symmetric (asymmetric) if this rate is (not) the mid-point of the corridor.[33] By changing the width of a symmetric interest corridor, the central bank may influence bank loan supply without changing its policy rate i^{RO}. As i^{RO} remains unchanged, certain marginal liquidity costs of granting loans do not alter. A potential effect of this monetary policy impulse on bank loan supply must thus be due to a change in expected marginal uncertainty costs. In Equilibrium I, banks never use the facilities to balance their uncertain liquidity needs. Changing the width of the corridor has therefore no effect on their loan supply decision. However, in Equilibria II and III, banks expect to use at least one of the facilities. In these equilibria widening the interest corridor has a contractionary effect on bank loan supply.

In Equilibrium II, an increase in the corridor width leads to higher marginal costs of holding positive as well as negative precautionary liquidity, see FOCs (24) and (25), so

[33] In the literature, the symmetry/asymmetry of the interest corridor also refers to the central bank's target interbank rate (Bindseil, 2014, chapters 4 and 13). With persisting high levels of excess liquidity the target rate is the deposit facility rate and, independent of the position of the refinancing rate, the interest corridor is considered to be asymmetric.

that banks reduce their holdings of this liquidity. This increases the expected marginal uncertainty costs of granting loans, and banks reduce their loan supply.[34] Due to the reduced holdings of precautionary liquidity banks expect to use the interbank market more intensively. An increase in γ hence amplifies the contractionary effect of a corridor widening on bank loan supply. Furthermore, the reduced holdings of precautionary liquidity are more significant, the higher the extent of uncertainty is, so that the effect of this monetary policy impulse on bank loan supply increases in χ.

In Equilibrium III, banks do not adjust their precautionary liquidity holdings relative to their loan supply, i.e., $\bar{t}^{*III}$ remains unchanged, as the symmetry of the interest corridor implies for all possible facility rates $G(\bar{t}^*) = 1 - G(\bar{t}^*) = 0.5$. Only in this case, the expected marginal costs equal the expected marginal revenues of holding precautionary liquidity, see FOC (27). Consequently, if the corridor is widened, the expected marginal uncertainty costs of granting loans will change only because of the marginal uncertainty costs $(i^{LF} - i^{DF})$ will increase. As the expected deficit and the expected surplus increase in the extent of uncertainty, the weight of these costs $\phi(\bar{t}^{*III})$ becomes larger, so that the impact of this monetary policy impulse on bank loan supply also increases in the extent of uncertainty.

6.4 Change in the Asymmetry of the Interest Corridor

There are several ways to implement an asymmetric interest corridor or to change an already existing asymmetry of the corridor. Here, we take a closer look at the consequences of a sole change in the deposit rate i^{DF}. Building on this analysis, we will draw some general conclusions with respect to the consequences of a change in the asymmetry of the interest corridor on bank loan supply at the end of this section. Changing only i^{DF} leads to

[34] If the non-negativity constraint on RO_b binds, banks will not be able to change their holdings of negative precautionary liquidity. The monetary policy impulse will have no effect on bank loan supply.

Result 5: *Changing only the deposit rate i^{DF} ($di^{RO} = di^{LF} = 0$), we obtain*

$$\frac{\partial L^{*I}}{\partial i^{DF}} = 0, \qquad \frac{\partial L^{*II}}{\partial i^{DF}} = \begin{cases} \frac{(1-c)\chi \bar{t}^{*II}}{\lambda} > 0 & \textit{if} \quad \bar{t}^{*II} > 0 \\ 0 & \textit{if} \quad \bar{t}^{*II} < 0 \end{cases}, \qquad \frac{\partial L^{*III}}{\partial i^{DF}} \begin{array}{l} = \frac{\phi(\bar{t}^{*III})}{\lambda} \\ + \frac{(1-c)\chi \bar{t}^{*III} G(\bar{t}^{*III})}{\lambda} > 0, \end{array}$$

$$\frac{\partial^2 L^{*I}}{\partial i^{DF} \partial \chi} = 0, \quad \frac{\partial^2 L^{*II}}{\partial i^{DF} \partial \chi} \begin{cases} = \frac{(1-c)\bar{t}^{*II}}{\lambda} > 0 & \textit{if} \quad \bar{t}^{*II} > 0 \\ = 0 & \textit{if} \quad \bar{t}^{*II} < 0 \end{cases}, \qquad \frac{\partial^2 L^{*III}}{\partial i^{DF} \partial \chi} \begin{array}{l} = \frac{\phi(\bar{t}^{*III})}{\lambda \chi} \\ + \frac{(1-c)\chi \bar{t}^{*III} G(\bar{t}^{*III})}{\lambda \chi} > 0, \end{array}$$

$$\frac{\partial^2 L^{*I}}{\partial i^{DF} \partial \gamma} = 0, \quad \frac{\partial^2 L^{*II}}{\partial i^{DF} \partial \gamma} \begin{cases} = \frac{(1-c)\chi G(\bar{t}^{*II})}{\lambda \gamma g(\bar{t}^{*II})} > 0 & \textit{if} \quad \bar{t}^{*II} > 0 \\ = 0 & \textit{if} \quad \bar{t}^{*II} < 0 \end{cases}, \quad \frac{\partial^2 L^{*III}}{\partial i^{DF} \partial \gamma} = 0.$$

Proof: See appendix.

A decrease in i^{DF} has either no or a contractionary effect on bank loan supply. In Equilibrium I and in Equilibrium II for $\bar{t}^{*II} < 0$, banks do not (expect to) use the deposit facility. A change in i^{DF} therefore has no effect on bank loan supply.

If banks hold positive precautionary liquidity in Equilibrium II ($\bar{t}^{*II} > 0$), a decrease in i^{DF} will have a contractionary effect on bank loan supply. As holding positive precautionary liquidity will become more expensive, see FOC (24), banks will reduce their holdings of this liquidity. The expected marginal uncertainty costs of granting loans will increase, and banks will reduce their loan supply. Analogous to the monetary policy impulse of a widening of the interest corridor, this policy impulse is more pronounced the higher the interbank market transaction costs and the extent of uncertainty are.

In Equilibrium III, a decrease in i^{DF} has a contractionary effect on bank loan supply. The driving force is the increase in the marginal uncertainty costs of granting loans ($i^{LF} - i^{DF}$). This direct effect is reflected by the first term of $\partial L^{*III}/\partial i^{DF}$. If banks hold positive precautionary liquidity ($\bar{t}^* > 0$), this contractionary effect will be reinforced. The decrease in i^{DF} increases the expected marginal costs of holding positive precautionary liquidity, so that banks will reduce their holdings of this liquidity. The resulting increase in the expected marginal uncertainty costs has a contractionary effect on bank loan supply (see second term of $\partial L^{III*}/\partial i^{DF}$). If banks hold negative precautionary liquidity ($\bar{t}^* < 0$) or no precautionary liquidity ($\bar{t}^* = 0$), the contractionary effect will be weakened as the expected marginal revenues of holding this liquidity will become higher so that banks will increase or start to hold negative precautionary liquidity which would have a negative

effect on the expected marginal uncertainty costs of granting loans.[35] The direct effect of this monetary policy impulse on the expected marginal uncertainty costs (increase in $i^{LF} - i^{DF}$) is stronger than the indirect effect (change in holdings of precautionary liquidity). As marginal uncertainty costs are more important to banks the higher the extent of uncertainty is (formally, $\phi(\bar{t}^*)$ increases in χ), the effect of this policy impulse is more pronounced the higher the extent of uncertainty is.

From this analysis we can conclude the following general results with respect to the effect of a change in the asymmetry of the interest corridor on bank loan supply. In Equilibrium I, changes in the asymmetry of the corridor do not have an impact on bank loan supply. In Equilibrium II, changes in the asymmetry of the corridor resulting in an increase in the costs of holding precautionary liquidity have a negative effect on bank loan supply. In Equilibrium III, those changes in the asymmetry of the corridor which leads to an increase in the spread between i^{LF} and i^{DF} have a negative effect on bank loan supply. Precautionary holdings of liquidity reinforce or dampen this effect.

6.5 Collateralization and Minimum Reserve Requirements

In our model, we have neglected two main elements of the ECB's operational framework: the collateralization of central bank credits and the minimum reserve requirements. Considering these elements is not crucial for our analysis as they only change our results quantitatively.

If we took the collateralization of central bank credits into consideration, banks would face opportunity costs of holding collateral. As a result, the expected marginal liquidity costs would increase.[36] Bank loan supply would be lower but our model results would not change qualitatively.

Reserve requirements lead to a structural liquidity deficit of the banking sector. In our model, such a deficit is already captured by considering cash withdrawals. Introducing reserve requirements would therefore simply increase the existing structural deficit. A

[35] This weakening effect will not exist if the non-negativity constraint on RO_b is binding, as then, banks would not be able to increase their negative holdings of precautionary liquidity any further. Formally, we have in this case $\partial L^{*III}/\partial i^{DF} = \phi(\underline{t})/\lambda > 0$ and $\partial^2 L^{*III}/\partial i^{DF}\partial\chi = (\phi(\underline{t}) - \underline{t}g(\underline{t})c)/\lambda\chi > 0$, with $\underline{t} = -c/((1-c)\chi) < 0$ (see proof of Lemma 1 in the appendix).

[36] For a respective analysis see, for example, Neyer and Wiemers (2004) and Berentsen and Monnet (2008).

main feature of the Eurosystem's minimum reserve system is that banks can make use of the averaging provision of required reserves during the reserve maintenance period. This allows banks to smooth out liquidity fluctuations. In our model, the marginal uncertainty costs of granting loans would decrease. Although this would have a positive effect on bank loan supply, the qualitative results of our model would not change.

7 Conclusion

The interbank market plays a crucial role for monetary policy implementation as it serves as the starting point of the monetary policy transmission mechanism. Based on a theoretical model, this paper analyzes how far interbank market frictions in the form of broadly defined transaction costs may impede the transmission mechanism. We find that interbank market frictions as well as bank idiosyncratic liquidity risk have a negative impact on bank loan supply but that they are not an impediment for the monetary transmission mechanism. The central bank can still influence commercial banks' expected liquidity costs and thus their loan supply. First, the central bank's standing facilities offer an alternative to using the friction-burdened and thus costly interbank market. Second, the rates on the facilities determine the costs of friction-induced holdings of precautionary liquidity. Increasing the rate on the deposit facility and decreasing the rate on the lending facility are expansionary monetary policy measures. Note that the sign of the central bank's interest rates does not play a role for this result.

Our model allows us to draw some conclusions for bank loan supply in the euro area and the ECB's monetary policy. During the financial crisis, interbank market frictions became stronger which made interbank trading more expensive. Furthermore, bank idiosyncratic liquidity risk increased. Both leads to a rise in banks' expected liquidity costs which has a negative impact on bank loan supply. However, from October 2008, when the collapse of the investment bank Lehman Brothers marked a peak of the global financial crisis, up to now (May 2017) the ECB has not only decreased its main refinancing rate from 4,75% to a historically low level of 0%, but it has also narrowed the interest corridor of its standing facilities from 200 basis points to 65 basis points. Thus, the ECB has not only significantly reduced banks' expected liquidity costs by lowering the price for liquidity borrowed directly

from the central bank, but in addition, it has reduced interbank market friction-induced liquidity costs. Consequently, monetary policy has been not only extremely expansive, but it has also made sure that interbank market frictions have not been a significant impediment for its pass-through to bank loan supply. Nevertheless, bank lending in the euro area, especially in the periphery, has been relatively weak. This indicates that there have been structural problems, such as insufficient bank capital or the lack of competitive projects in need of financing.

Finally, it should be noted that in our model, the banking sector as a whole faces a structural liquidity deficit forcing banks to borrow liquidity from the central bank. However, in March 2015, the Eurosystem started its public sector purchase program. The huge amount of government bond purchases implied that, beginning of August 2015, the banking sector as a whole faced a structural liquidity surplus instead of a deficit.[37] Both, a deep analysis of commercial banks' liquidity management in an environment of a structural liquidity surplus and in the presence of heterogeneous agents goes beyond the scope of this paper and presents an interesting issue for future research.

A Appendix

A.1 Proof of Lemma 1

It follows from (2) to (4), that the first stage optimization problem of bank b reads:

$$\begin{aligned}\max_{L_b\in\mathbb{R}_0^+, RO_b\in\mathbb{R}_0^+} f(L_b, RO_b) = {} & i^L L_b - \tfrac{1}{2}\lambda L_b^2 - i^{RO} RO_b \\ & - \max\left\{i^{IBM} - \gamma, i^{DF}\right\} \int_{t^{min}}^{\bar{t}_b} N_b g(t_b)\, dt_b \\ & - \min\left\{i^{IBM} + \gamma, i^{LF}\right\} \int_{\bar{t}_b}^{t^{max}} N_b g(t_b)\, dt_b,\end{aligned} \tag{29}$$

with N_b given by (6) and $\bar{t}_b$ by (7).

[37] Note that since the beginning of the ECB's full allotment policy in 2008, the banking sector operates in an environment of excess liquidity. However, the difference between the period until August 2015 and the period thereafter is that since August 2015, the banking sector has no longer been able to reduce excess liquidity by decreasing its borrowing from the ECB. Even if no bank borrows from the ECB, there will be excess liquidity.

By applying Leibniz's rule and making use of the fact that $N_b = 0$ for $t_b = \bar{t}_b$, we obtain:

$$\begin{aligned} f'_{RO_b}(L_b, R_b) = -i^{RO} - \max\left\{i^{IBM} - \gamma, i^{DF}\right\} \int_{t^{min}}^{\bar{t}_b} \tfrac{\partial N_b}{\partial RO_b} g(t_b) dt_b \\ - \min\left\{i^{IBM} + \gamma, i^{LF}\right\} \int_{\bar{t}_b}^{t^{max}} \tfrac{\partial N_b}{\partial RO_b} g(t_b) dt_b. \end{aligned} \tag{30}$$

We can infer from (6) that $\frac{\partial N_b}{\partial RO_b} = -1$. Insertion of this in (30) and rewriting terms yields

$$\begin{aligned} f'_{RO_b}(L_b, R_b) = -i^{RO} + \max\left\{i^{IBM} - \gamma, i^{DF}\right\} G\left(\bar{t}_b\right) \\ + \min\left\{i^{IBM} + \gamma, i^{LF}\right\} \left[1 - G\left(\bar{t}_b\right)\right]. \end{aligned} \tag{31}$$

Note that $f'_{RO_b}(L_b, R_b)$ decreases in $G\left(\bar{t}_b\right) \in [0, 1]$, which in turn (weakly) increases in $\bar{t}_b$. Moreover, we know from (7) that

- $\bar{t}_b$ increases in RO_b, so that $f'_{RO_b}(L_b, R_b)$ (weakly) decreases in RO_b,
- and from the restriction $RO_b \geq 0$ that $\bar{t}_b$ is restricted to $\bar{t}_b \geq -\frac{c}{(1-c)\chi} =: \underline{t}$.

Denoting optima by the superscript *opt*, we can distinguish three cases:

1. If $i^{IBM} > i^{RO} + \gamma$, then $f'_{RO_b}(L_b, R_b) > 0$ for all $RO_b < \infty$. Therefore, we obtain $RO_b^{opt} \to \infty$.
2. If $i^{IBM} \in \left[i^{RO} - \gamma, i^{RO} + \gamma\right]$, the first order conditions for optimal borrowing from the refinancing operations are

$$f'_{RO_b}(L_b^{opt}, R_b^{opt}) \leq 0, \quad f'_{RO_b}(L_b^{opt}, R_b^{opt}) \cdot RO_b^{opt} = 0, \quad RO_b^{opt} \geq 0. \tag{32}$$

   Accordingly, if the non-negativity constraint for RO_b does not bind, marginal costs and expected marginal revenues of borrowing from the refinancing operations will be balanced, $f'_{RO_b}(L_b^{opt}, R_b^{opt}) = 0$. However, if for $RO_b = 0$, i.e. for $\bar{t}_b = \underline{t}$, expected marginal revenues are strictly lower than marginal costs, the non-negativity constraint becomes binding, further decreases in RO_b are not possible. Then, $RO_b^{opt} = 0$ and $f'_{RO_b}(L_b^{opt}, R_b^{opt}) < 0$.

3. If $i^{IBM} < i^{RO} - \gamma$, then $f'_{RO_b}(L_b, R_b) < 0$ for all $RO_b \geq 0$. The non-negativity constraint on RO_b binds, so that $RO_b^{opt} = 0$.

Consequently, we have shown that if $i^{IBM*} \in \left[i^{RO} - \gamma, i^{RO} + \gamma\right]$ and if the non-negativity constraint for RO_b does not bind, a bank's optimal borrowing from the refinancing operations is described by (8). ■

A.2 Proof of Lemma 3

By applying Leibniz's rule on (29) and making use of the property that $N_b = 0$ for $t_b = \bar{t}_b$, we obtain:

$$\begin{aligned} f'_{L_b}(L_b, R_b) = i^L - \lambda L_b - \max\left\{i^{IBM} - \gamma, i^{DF}\right\} \int_{t^{min}}^{\bar{t}_b} \tfrac{\partial N_b}{\partial L_b} g(t_b) dt_b \\ - \min\left\{i^{IBM} + \gamma, i^{LF}\right\} \int_{\bar{t}_b}^{t^{max}} \tfrac{\partial N_b}{\partial L_b} g(t_b) dt_b. \end{aligned} \tag{33}$$

We can infer from (6) that $\frac{\partial N_b}{\partial L_b} = c + (1-c)\,\chi t_b$. Insertion of this in (33) and rewriting terms yields

$$\begin{aligned} f'_{L_b}(L_b, R_b) = i^L - \lambda L_b \\ - c \max\left\{i^{IBM} - \gamma, i^{DF}\right\} G\left(\bar{t}_b\right) - c \min\left\{i^{IBM} + \gamma, i^{LF}\right\} \left[1 - G\left(\bar{t}_b\right)\right] \\ - (1-c)\,\chi \max\left\{i^{IBM} - \gamma, i^{DF}\right\} \int_{t^{min}}^{\bar{t}_b} t_b g(t_b) dt_b \\ - (1-c)\,\chi \min\left\{i^{IBM} + \gamma, i^{LF}\right\} \int_{\bar{t}_b}^{t^{max}} t_b g(t_b) dt_b. \end{aligned} \tag{34}$$

Considering (8) and that due to $E[t_b] = 0$ we have $-\int_{t^{min}}^{\bar{t}_b^{opt}} t_b g(t_b)\, dt_b = \int_{\bar{t}_b^{opt}}^{t^{max}} t_b g(t_b)\, dt_b$, (13) is thus the first order condition for optimal lending to the non-banking sector. ■

A.3 Proof of Result 1

Equilibrium I

Considering Lemma 3, Proposition 2 and (23), L^{*I} is implicitly defined by

$$H(\cdot)^I := -i^L + \lambda L^{*I} + ci^{RO} + \phi(0)2\gamma, \tag{35}$$

$$\text{where} \quad \phi(0) = (1-c)\chi \int_0^{t^{max}} t_b g(t_b)\, dt_b,$$

$$\text{so that} \quad \frac{\partial H(\cdot)^I}{\partial \gamma} = 2\phi(0) \tag{36}$$

$$\text{and} \quad \frac{\partial H(\cdot)^I}{\partial L^{*I}} = \lambda. \tag{37}$$

Applying the implicit function theorem yields

$$\frac{\partial L^{*I}}{\partial \gamma} = -\frac{\frac{\partial H(\cdot)^I}{\partial \gamma}}{\frac{\partial H(\cdot)^I}{\partial L^{*I}}} = -\frac{2\phi(0)}{\lambda} < 0. \tag{38}$$

■

Equilibrium II

Considering Lemma 3, Proposition 3 and (26), L^{*II} is implicitly defined by

$$H(\cdot)^{II} := -i^L + \lambda L^{*II} + ci^{RO} + \phi(\bar{t}^{*II})2\gamma, \tag{39}$$

$$\text{where} \quad \phi(\bar{t}^{*II}) = (1-c)\chi \int_{\bar{t}^{*II}}^{t^{max}} t_b g(t_b)\, dt_b, \tag{40}$$

$$\text{and} \quad \bar{t}^{*II} = \frac{RO^{*II} - cL^{*II}}{(1-c)\chi L^{*II}}, \tag{41}$$

$$\text{so that} \quad \frac{\partial H(\cdot)^{II}}{\partial \gamma} = 2\phi(\bar{t}^{*II}) - 2\gamma(1-c)\chi \bar{t}^{*II} g(\bar{t}^{*II}) \frac{\partial \bar{t}^{*II}}{\partial \gamma}, \tag{42}$$

$$\text{with} \quad \frac{\partial \bar{t}^{*II}}{\partial \gamma} = \frac{\partial \bar{t}^{*II}}{\partial RO^{*II}} \frac{\partial RO^{*II}}{\partial \gamma} = \frac{1}{(1-c)\chi L^{*II}} \frac{\partial RO^{*II}}{\partial \gamma}, \tag{43}$$

$$\text{and} \quad \frac{\partial H(\cdot)^{II}}{\partial L^{*II}} = \lambda - 2\gamma(1-c)\chi \bar{t}^{*II} g(\bar{t}^{*II}) \frac{\partial \bar{t}^{*II}}{\partial L^{*II}}, \tag{44}$$

$$\text{with} \quad \frac{\partial \bar{t}^{*II}}{\partial L^{*II}} = \frac{\partial \bar{t}^{*II}}{\partial RO^{*II}} \frac{\partial RO^{*II}}{\partial L^{*II}} = \frac{\left(\frac{\partial RO^{*II}}{\partial L^{*II}} - c\right) L^{*II} - (RO^{*II} - cL^{*II})}{(1-c)\chi \left(L^{*II}\right)^2}. \tag{45}$$

Considering Lemma 1 and Proposition 3, RO^{*II} is implicitly defined by

$$Z(\cdot)^{II} := \begin{cases} -i^{RO} + i^{DF} + 2\gamma \int_{\bar{t}^{*II}}^{t^{max}} g(t_b)\, dt_b & \text{if} \quad \bar{t}^{*II} > 0, \\ -i^{RO} + i^{LF} - 2\gamma \int_{t^{min}}^{\bar{t}^{*II}} g(t_b)\, dt_b & \text{if} \quad \bar{t}^{*II} < 0, \end{cases} \tag{46}$$

$$\text{so that} \quad \frac{\partial Z(\cdot)^{II}}{\partial \gamma} = \begin{cases} 2(1 - G(\bar{t}^{*II})) & \text{if} \quad \bar{t}^{*II} > 0, \\ -2G(\bar{t}^{*II}) & \text{if} \quad \bar{t}^{*II} < 0, \end{cases} \tag{47}$$

$$\frac{\partial Z(\cdot)^{II}}{\partial L^{*II}} = \frac{2\gamma RO^{*II} g(\bar{t}^{*II})}{(1-c)\chi (L^{*II})^2} \text{ for } \bar{t}^{*II} \gtreqless 0, \tag{48}$$

$$\frac{\partial Z(\cdot)^{II}}{\partial RO^{*II}} = -\frac{2\gamma g(\bar{t}^{*II})}{(1-c)\chi L^{*II}} \text{ for } \bar{t}^{*II} \gtreqless 0. \tag{49}$$

Applying the implicit function theorem yields

$$\frac{\partial RO^{*II}}{\partial \gamma} = -\frac{\frac{\partial Z(\cdot)^{II}}{\partial \gamma}}{\frac{\partial Z(\cdot)^{II}}{\partial RO^{*II}}} = \begin{cases} \frac{2(1-G(\bar{t}^{*II}))(1-c)\chi L^{*II}}{2\gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} > 0, \\ -\frac{2G(\bar{t}^{*II})(1-c)\chi L^{*II}}{2\gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} < 0, \end{cases} \tag{50}$$

$$\frac{\partial RO^{*II}}{\partial L^{*II}} = -\frac{\frac{\partial Z(\cdot)^{II}}{\partial L^{*II}}}{\frac{\partial Z(\cdot)^{II}}{\partial RO^{*II}}} = \frac{RO^{*II}}{L^{*II}}, \tag{51}$$

which implies that (43) and (45) simplify to

$$\frac{\partial \bar{t}^{*II}}{\partial \gamma} = \begin{cases} \frac{1-G(\bar{t}^{*II})}{\gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} > 0, \\ -\frac{G(\bar{t}^{*II})}{\gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} < 0, \end{cases} \tag{52}$$

$$\frac{\partial \bar{t}^{*II}}{\partial L^{*II}} = 0, \tag{53}$$

which again means that (42) and (44) become

$$\frac{\partial H(\cdot)^{II}}{\partial \gamma} = \begin{cases} 2\phi(\bar{t}^{*II}) - (1-c)\chi \bar{t}^{*II} 2(1 - G(\bar{t}^{*II})) & \text{if} \quad \bar{t}^{*II} > 0, \\ 2\phi(\bar{t}^{*II}) + (1-c)\chi \bar{t}^{*II} 2G(\bar{t}^{*II}) & \text{if} \quad \bar{t}^{*II} < 0, \end{cases} \tag{54}$$

$$\frac{\partial H(\cdot)^{II}}{\partial L^{*II}} = \lambda. \tag{55}$$

Applying the implicit function theorem yields

$$\frac{\partial L^{*II}}{\partial \gamma} = -\frac{\frac{\partial H(\cdot)^{II}}{\partial \gamma}}{\frac{\partial H(\cdot)^{II}}{\partial L^{*II}}} = \left\{ \begin{array}{ll} -\frac{1}{\lambda}\left[2\phi(\bar{t}^{*II}) - 2(1-c)\chi\bar{t}^{*II}(1-G(\bar{t}^{*II}))\right] & \text{if} \quad \bar{t}^{*II} > 0 \\ -\frac{1}{\lambda}\left[2\phi(\bar{t}^{*II}) + 2(1-c)\chi\bar{t}^{*II}G(\bar{t}^{*II})\right] & \text{if} \quad \bar{t}^{*II} < 0 \end{array} \right\} < 0. \tag{56}$$

With respect to the negative sign of $\partial L^{*II}/\partial\gamma$ we have to show that the terms in square brackets are positive:

$$\begin{aligned} & 2\phi(\bar{t}^{*II}) - 2(1-c)\chi\bar{t}^{*II}(1-G(\bar{t}^{*II})) > 0 \\ & \Leftrightarrow \int_{\bar{t}^{*II}}^{t^{max}} t_b g(t_b)\, dt_b - \bar{t}^{*II}(1-G(\bar{t}^{*II}) > 0 \\ & \Leftrightarrow \frac{\int_{\bar{t}^{*}}^{t^{max}} t_b g(t_b)\, dt_b}{1-G(\bar{t}^{*II})} > \bar{t}^{*II} \\ & \Leftrightarrow E[t_b|t_b > \bar{t}^{*II}] > \bar{t}^{*II} \end{aligned}$$

$$\begin{aligned} & 2\phi(\bar{t}^{*II}) + 2(1-c)\chi\bar{t}^{*II}G(\bar{t}^{*II}) > 0 \\ & \Leftrightarrow \int_{\bar{t}^{*II}}^{t^{max}} t_b g(t_b)\, dt_b + \bar{t}^{*II}G(\bar{t}^{*II}) > 0 \\ & \Leftrightarrow -\int_{t^{min}}^{\bar{t}^{*II}} t_b g(t_b)\, dt_b + \bar{t}^{*II}G(\bar{t}^{*II}) > 0 \\ & \Leftrightarrow \bar{t}^{*II} > \frac{\int_{t^{min}}^{\bar{t}^{*II}} t_b g(t_b)\, dt_b}{G(\bar{t}^{*II})} \\ & \Leftrightarrow \bar{t}^{*II} > E[t_b|t_b < \bar{t}^{*II}] \end{aligned}$$

As $E[t_b|t_b > \bar{t}^{*II}] > \bar{t}^{*II}$ and $\bar{t}^{*II} > E[t_b|t_b < \bar{t}^{*II}]$, the terms in square brackets are positive, so that $\partial L^{*II}/\partial\gamma > 0$. Note with respect to the second transformation shown in the right hand column that due to $E[t_b] = 0$, we have that $\int_{\bar{t}^{*II}}^{t^{max}} t_b g(t_b)\, dt_b = -\int_{t^{min}}^{\bar{t}^{*II}} t_b g(t_b)\, dt_b$. ■

Equilibrium III

Trivial, inactive interbank market.

A.4 Proof of Result 2

Equilibrium I

Partially differentiating $H(\cdot)^I$ given by (35) w.r.t. χ, we get

$$\frac{\partial H(\cdot)^I}{\partial \chi} = (1-c)\int_0^{t^{max}} t_b g(t_b)\, dt_b 2\gamma = \frac{\phi(0)2\gamma}{\chi}. \tag{57}$$

Considering (37) and (57), applying the implicit function theorem yields

$$\frac{\partial L^{*I}}{\partial \chi} = -\frac{\frac{\partial H(\cdot)^I}{\partial \chi}}{\frac{\partial H(\cdot)^I}{\partial L^{*I}}} = -\frac{\phi(0)2\gamma}{\lambda\chi} < 0. \tag{58}$$

■

Equilibrium II

Partially differentiating $H(\cdot)^{II}$ given by (39) w.r.t. χ, we get

$$\frac{\partial H(\cdot)^{II}}{\partial \chi} = (1-c)2\gamma \left(\int_{\bar{t}^{*II}}^{t^{max}} t_b g(t_b)\, dt_b - \chi \bar{t}^{*II} g(\bar{t}^{*II}) \frac{\partial \bar{t}^{*II}}{\partial \chi} \right), \tag{59}$$

$$\text{where} \quad \frac{\partial \bar{t}^{*II}}{\partial \chi} = \frac{\partial \bar{t}^{*II}}{\partial RO^{*II}} \frac{\partial RO^{*II}}{\partial \chi} = \frac{\frac{\partial RO^{*II}}{\partial \chi}\chi - (RO^{*II} - cL^{*II})}{(1-c)L^{*II}\chi^2}. \tag{60}$$

Due to

$$\frac{\partial Z(\cdot)^{II}}{\partial \chi} = \frac{g(\bar{t}^{*II})\bar{t}^{*II}2\gamma}{\chi} \text{ for } \bar{t}^{*II} \gtreqless 0 \tag{61}$$

and (49), applying the implicit function theorem yields

$$\frac{\partial RO^{*II}}{\partial \chi} = -\frac{\frac{\partial Z(\cdot)^{II}}{\partial \chi}}{\frac{\partial Z(\cdot)^{II}}{\partial RO^{*II}}} = \bar{t}^{*II}(1-c)L^{*II}, \tag{62}$$

so that (60) becomes

$$\frac{\partial \bar{t}^{*II}}{\partial \chi} = 0, \tag{63}$$

and (59) simplifies to

$$\frac{\partial H(\cdot)^{II}}{\partial \chi} = \frac{\phi(\bar{t}^{*II})2\gamma}{\chi}. \tag{64}$$

Considering (55) and (64), applying the implicit function theorem yields

$$\frac{\partial L^{*II}}{\partial \chi} = -\frac{\frac{\partial H(\cdot)^{II}}{\partial \chi}}{\frac{\partial H(\cdot)^{II}}{\partial L^{*II}}} = -\frac{\phi(\bar{t}^{*II})2\gamma}{\lambda\chi} < 0. \tag{65}$$

Equilibrium III

Considering Lemma 3, Proposition 4 and (28), L^{*III} is implicitly defined by

$$H(\cdot)^{III} := -i^L + \lambda L^{*III} + ci^{RO} + \phi(\bar{t}^{*III})(i^{LF} - i^{DF}), \tag{66}$$

$$\text{where} \quad \phi(\bar{t}^{*III}) = (1-c)\chi \int_{\bar{t}^{*III}}^{t^{max}} t_b g(t_b)\, dt_b \tag{67}$$

$$\text{and} \quad \bar{t}^{*III} = \frac{RO^{*III} - cL^{*III}}{(1-c)\chi L^{*III}}, \tag{68}$$

$$\text{so that} \quad \frac{\partial H(\cdot)^{III}}{\partial \chi} = (1-c)(i^{LF} - i^{DF}) \left(\int_{\bar{t}^{*III}}^{t^{max}} t_b g(t_b)\, dt_b - \chi \bar{t}^{*II} g(\bar{t}^{*III}) \frac{\partial \bar{t}^{*III}}{\partial \chi} \right), \tag{69}$$

$$\text{with} \quad \frac{\partial \bar{t}^{*III}}{\partial \chi} = \frac{\partial \bar{t}^{*III}}{\partial RO^{*III}} \frac{\partial RO^{*III}}{\partial \chi} = \frac{\frac{\partial RO^{*III}}{\partial \chi}\chi - (RO^{*III} - cL^{*III})}{(1-c)L^{*III}\chi^2}, \tag{70}$$

$$\text{and} \quad \frac{\partial H(\cdot)^{III}}{\partial L^{*III}} = \lambda - (i^{LF} - i^{DF})(1-c)\chi \bar{t}^{*III} g(\bar{t}^{*III}) \frac{\partial \bar{t}^{*III}}{\partial L^{*III}}, \tag{71}$$

$$\text{with} \quad \frac{\partial \bar{t}^{*III}}{\partial L^{*III}} = \frac{\partial \bar{t}^{*III}}{\partial RO^{*III}} \frac{\partial RO^{*III}}{\partial L^{*III}} = \frac{\left(\frac{\partial RO^{*III}}{\partial L^{*III}} - c\right) L^{*III} - (RO^{*III} - cL^{*III})}{(1-c)\chi \left(L^{*III}\right)^2}. \tag{72}$$

Considering Lemma 1 and Proposition 4, RO^{*III} is implicitly defined by

$$Z(\cdot)^{III} := -i^{RO} + i^{DF} \int_{t^{min}}^{\bar{t}^{*III}} g(t_b)\, dt_b + i^{LF} \int_{\bar{t}^{*III}}^{t^{max}} g(t_b)\, dt_b, \tag{73}$$

$$\text{so that} \quad \frac{\partial Z(\cdot)^{III}}{\partial \chi} = \frac{(i^{LF} - i^{DF}) g(\bar{t}^{*II}) \bar{t}^{*III}}{\chi}, \tag{74}$$

$$\frac{\partial Z(\cdot)^{III}}{\partial L^{*III}} = \frac{(i^{LF} - i^{DF}) RO^{*III} g(\bar{t}^{*III})}{(1-c)\chi (L^{*II})^2}, \tag{75}$$

$$\frac{\partial Z(\cdot)^{III}}{\partial RO^{*III}} = -\frac{(i^{LF} - i^{DF}) g(\bar{t}^{*III})}{(1-c)\chi L^{*III}}. \tag{76}$$

Applying the implicit function theorem yields

$$\frac{\partial RO^{*III}}{\partial \chi} = \bar{t}^{*III}(1-c)L^{*III}, \tag{77}$$

$$\frac{\partial RO^{*III}}{\partial L^{*III}} = -\frac{\frac{\partial Z(\cdot)^{III}}{\partial L^{*III}}}{\frac{\partial Z(\cdot)^{III}}{\partial RO^{*III}}} = \frac{RO^{*III}}{L^{*III}}, \tag{78}$$

implying that (70) and (72) simplify to

$$\frac{\partial \bar{t}^{*III}}{\partial \chi} = 0, \tag{79}$$

$$\frac{\partial \bar{t}^{*III}}{\partial L^{*III}} = 0, \tag{80}$$

so that (69)and (71) become

$$\frac{\partial H(\cdot)^{III}}{\partial \chi} = \frac{\phi(\bar{t}^{*III})(i^{LF} - i^{DF})}{\chi}, \tag{81}$$

$$\frac{\partial H(\cdot)^{III}}{\partial L^{*III}} = \lambda. \tag{82}$$

Applying the implicit function theorem yields

$$\frac{\partial L^{*III}}{\partial \chi} = -\frac{\frac{\partial H(\cdot)^{III}}{\partial \chi}}{\frac{\partial H(\cdot)^{III}}{\partial L^{*III}}} = -\frac{\phi(\bar{t}^{*III})(i^{LF} - i^{DF})}{\chi\lambda} < 0. \tag{83}$$

■

A.5 Proof of Result 3

Equilibrium I

Partially differentiating $H(\cdot)^I$ given by (35) w.r.t. i^{RO}, we get

$$\frac{\partial H(\cdot)^I}{\partial i^{RO}} = c. \tag{84}$$

Considering (37) and (84), applying the implicit function theorem yields

$$\frac{\partial L^{*I}}{\partial i^{RO}} = -\frac{\frac{\partial H(\cdot)^I}{\partial i^{RO}}}{\frac{\partial H(\cdot)^I}{\partial L^{*I}}} = -\frac{c}{\lambda} < 0. \tag{85}$$

■

Equilibrium II

Partially differentiating $H(\cdot)^{II}$ given by (39) w.r.t. i^{RO}, we get

$$\frac{\partial H(\cdot)^{II}}{\partial i^{RO}} = c - 2\gamma(1-c)\chi\bar{t}^{*II}g(\bar{t}^{*II})\frac{\partial \bar{t}^{*II}}{\partial i^{RO}}, \tag{86}$$

$$\text{where} \quad \frac{\partial \bar{t}^{*II}}{\partial i^{RO}} = \frac{\partial \bar{t}^{*II}}{\partial RO^{*II}}\frac{\partial RO^{*II}}{\partial i^{RO}} = \frac{1}{(1-c)\chi L^{*II}}\frac{\partial RO^{*II}}{\partial i^{RO}}. \tag{87}$$

Partially differentiating $Z(\cdot)^{II}$ given by (46) w.r.t. i^{RO}, considering that $\frac{di^{DF}}{di^{RO}} = \frac{di^{LF}}{di^{RO}} = 1$, we get

$$\frac{\partial Z(\cdot)^{II}}{\partial i^{RO}} = 0 \quad \text{for } \bar{t}^{*II} \gtrless 0. \tag{88}$$

Considering (49) and (88), applying the implicit function theorem yields

$$\frac{\partial RO^{*II}}{\partial i^{RO}} = -\frac{\frac{\partial Z(\cdot)^{II}}{\partial i^{RO}}}{\frac{\partial Z(\cdot)^{II}}{\partial RO^{*II}}} = 0, \tag{89}$$

implying that (87) becomes

$$\frac{\partial \bar{t}^{*II}}{\partial i^{RO}} = 0, \tag{90}$$

so that (86) simplifies to

$$\frac{\partial H(\cdot)^{II}}{\partial i^{RO}} = c. \tag{91}$$

Considering (55) and (91), applying the implicit function theorem yields

$$\frac{\partial L^{*II}}{\partial i^{RO}} = -\frac{\frac{\partial H(\cdot)^{II}}{\partial i^{RO}}}{\frac{\partial H(\cdot)^{II}}{\partial L^{*II}}} = -\frac{c}{\lambda} < 0. \tag{92}$$

■

Equilibrium III

Due to $\frac{di^{DF}}{di^{RO}} = \frac{di^{LF}}{di^{RO}} = 1$ it follows from (66) that

$$\frac{\partial H(\cdot)^{III}}{\partial i^{RO}} = c - (i^{LF} - i^{DF})(1-c)\chi \bar{t}^{*III} g(\bar{t}^{*III}) \frac{\partial \bar{t}^{*III}}{\partial i^{RO}}, \tag{93}$$

$$\text{where} \quad \frac{\partial \bar{t}^{*III}}{\partial i^{RO}} * = \frac{\partial \bar{t}^{*III}}{\partial RO^{*III}} \frac{\partial RO^{*III}}{\partial i^{RO}} = \frac{1}{(1-c)\chi L^{*III}} \frac{\partial RO^{*III}}{\partial i^{RO}}, \tag{94}$$

and from (73) that

$$\frac{\partial Z(\cdot)^{III}}{\partial i^{RO}} = 0. \tag{95}$$

Considering (95), applying the implicit function theorem thus yields

$$\frac{\partial RO^{*III}}{\partial i^{RO}} = -\frac{\frac{\partial Z(\cdot)^{III}}{\partial i^{RO}}}{\frac{\partial Z(\cdot)^{III}}{\partial RO^{*III}}} = 0, \tag{96}$$

implying that (94) becomes

$$\frac{\partial \bar{t}^{*III}}{\partial i^{RO}} = 0, \tag{97}$$

and that (93) simplifies to

$$\frac{\partial H(\cdot)^{III}}{\partial i^{RO}} = c. \tag{98}$$

Considering (82), applying the implicit function theorem yields

$$\frac{\partial L^{*III}}{\partial i^{RO}} = -\frac{\frac{\partial H(\cdot)^{III}}{\partial i^{RO}}}{\frac{\partial H(\cdot)^{III}}{\partial L^{*III}}} = -\frac{c}{\lambda} < 0. \tag{99}$$

■

A.6 Proof of Result 4

Equilibrium I

Partially differentiating $H(\cdot)^I$ given by (35) w.r.t. i^{LF} considering that $\frac{di^{DF}}{di^{LF}} = -1$ we get

$$\frac{\partial H(\cdot)^I}{\partial i^{LF}} = 0. \tag{100}$$

Considering (37), applying the implicit function theorem thus yields

$$\frac{\partial L^{*I}}{\partial i^{LF}} = -\frac{\frac{\partial H(\cdot)^I}{\partial i^{LF}}}{\frac{\partial H(\cdot)^I}{\partial L^{*I}}} = 0. \tag{101}$$

■

Equilibrium II

Due to $\frac{di^{DF}}{di^{LF}} = -1$ it follows from (39) that

$$\frac{\partial H(\cdot)^{II}}{\partial i^{LF}} = -2\gamma(1-c)\chi\bar{t}^{*II}g(\bar{t}^{*II})\frac{\partial \bar{t}^{*II}}{\partial i^{LF}}, \tag{102}$$

$$\text{where} \quad \frac{\partial \bar{t}^{*II}}{\partial i^{LF}} = \frac{\partial \bar{t}^{*II}}{\partial RO^{*II}}\frac{\partial RO^{*II}}{\partial i^{LF}} = \frac{1}{(1-c)\chi L^{*II}}\frac{\partial RO^{*II}}{\partial i^{LF}}, \tag{103}$$

and from (46) that

$$\frac{\partial Z(\cdot)^{II}}{\partial i^{LF}} = \begin{cases} -1 & \text{if} \quad \bar{t}^{*II} > 0, \\ 1 & \text{if} \quad \bar{t}^{*II} < 0. \end{cases} \tag{104}$$

Considering (49) and (104), applying the implicit function theorem yields

$$\frac{\partial RO^{*II}}{\partial i^{LF}} = -\frac{\frac{\partial Z(\cdot)^{II}}{\partial i^{LF}}}{\frac{\partial Z(\cdot)^{II}}{\partial RO^{*II}}} = \begin{cases} -\frac{(1-c)\chi L^{*II}}{2\gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} > 0, \\ \frac{(1-c)\chi L^{*II}}{2\gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} < 0, \end{cases} \tag{105}$$

which means that (103) becomes

$$\frac{\partial \bar{t}^{*II}}{\partial i^{LF}} = \begin{cases} -\frac{1}{2\gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} > 0, \\ \frac{1}{2\gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} < 0, \end{cases} \tag{106}$$

so that (102) simplifies to

$$\frac{\partial H(\cdot)^{II}}{\partial i^{LF}} = \begin{cases} (1-c)\chi \bar{t}^{*II} & \text{if} \quad \bar{t}^{*II} > 0, \\ -(1-c)\chi \bar{t}^{*II} & \text{if} \quad \bar{t}^{*II} < 0. \end{cases} \tag{107}$$

Considering (55), applying the implicit function theorem thus yields

$$\frac{\partial L^{*II}}{\partial i^{LF}} = -\frac{\frac{\partial H(\cdot)^{II}}{\partial i^{LF}}}{\frac{\partial H(\cdot)^{II}}{\partial L^{*II}}} = \left\{ \begin{array}{ll} -\frac{(1-c)\chi \bar{t}^{*II}}{\lambda} & \text{if} \quad \bar{t}^{*II} > 0 \\ \frac{(1-c)\chi \bar{t}^{*II}}{\lambda} & \text{if} \quad \bar{t}^{*II} < 0 \end{array} \right\} \tag{108}$$

$$= -\frac{(1-c)\chi |\bar{t}^{*II}|}{\lambda} < 0 \text{ for } \bar{t}^{*II} \gtrless 0. \tag{109}$$

With respect to the cross derivatives we have

$$\frac{\partial^2 L^{*II}}{\partial i^{LF} \partial \chi} = \left\{ \begin{array}{ll} -\frac{1-c}{\lambda}\left[\bar{t}^{*II} + \chi \frac{\partial \bar{t}^{*II}}{\partial \chi}\right] & \text{if} \quad \bar{t}^{*II} > 0 \\ \frac{1-c}{\lambda}\left[\bar{t}^{*II} + \chi \frac{\partial \bar{t}^{*II}}{\partial \chi}\right] & \text{if} \quad \bar{t}^{*II} < 0 \end{array} \right\} = -\frac{(1-c)|\bar{t}^{*II}|}{\lambda} < 0, \tag{110}$$

$$\frac{\partial^2 L^{*II}}{\partial i^{LF} \partial \gamma} = \left\{ \begin{array}{ll} -\frac{(1-c)\chi}{\lambda}\frac{\partial \bar{t}^{*II}}{\partial \gamma} = -\frac{(1-c)\chi(1-G(\bar{t}^{*II})}{\lambda \gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} > 0 \\ \frac{(1-c)\chi}{\lambda}\frac{\partial \bar{t}^{*II}}{\partial \gamma} = -\frac{(1-c)\chi G(\bar{t}^{*II})}{\lambda \gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} < 0 \end{array} \right\} < 0. \tag{111}$$

Although RO^{*II} *and* L^{*II} change in χ (see (62) and (65)) and in γ (see (50) and (56)), the change in L^{*II} does not evoke a change in $\bar{t}^{*II}$ (see (53)) due to a respective change in RO^{*II} (see (51)). Consequently, with respect to $\frac{\partial \bar{t}^{*II}}{\partial \chi}$ $\frac{\partial \bar{t}^{*II}}{\partial \gamma}$ it will be sufficient to focus on $\frac{\partial \bar{t}^{*II}}{\partial RO^{*II}} \frac{\partial RO^{*II}}{\partial \chi}$ and $\frac{\partial \bar{t}^{*II}}{\partial RO^{*II}} \frac{\partial RO^{*II}}{\partial \gamma}$ given by (60), (63), (43) and (52).

■

Equilibrium III

Due to $\frac{di^{DF}}{di^{LF}} = -1$ it follows from (66) that

$$\frac{\partial H(\cdot)^{III}}{\partial i^{LF}} = (1-c)\chi \left[2\int_{\bar{t}^{*III}}^{t^{max}} t_b g(t_b)\, dt_b - (i^{LF} - i^{DF})\bar{t}^{*II} g(\bar{t}^{*III}) \frac{\partial \bar{t}^{*III}}{\partial i^{LF}} \right], \tag{112}$$

$$\text{where} \quad \frac{\partial \bar{t}^{*III}}{\partial i^{LF}} = \frac{\partial \bar{t}^{*III}}{\partial RO^{*III}} \frac{\partial RO^{*III}}{\partial i^{LF}} = \frac{1}{(1-c)\chi L^{*III}} \frac{\partial RO^{*III}}{\partial i^{LF}}, \tag{113}$$

and from (73) that

$$\frac{\partial Z(\cdot)^{III}}{\partial i^{LF}} = 1 - 2G(\bar{t}^{*III}). \tag{114}$$

Considering (76) and (114), applying the implicit function theorem yields

$$\frac{\partial RO^{*III}}{\partial i^{LF}} = -\frac{\frac{\partial Z(\cdot)^{III}}{\partial i^{LF}}}{\frac{\partial Z(\cdot)^{III}}{\partial RO^{*III}}} = \frac{(1-2G(\bar{t}^{*III}))(1-c)\chi L^{*III}}{(i^{LF} - i^{DF})g(\bar{t}^{*III})} = 0, \tag{115}$$

as due to the symmetric interest corridor $G(\bar{t}^{*III}) = 1 - G(\bar{t}^{*III}) = 0.5$ (see (27)). Therefore, (113) becomes

$$\frac{\partial \bar{t}^{*III}}{\partial i^{LF}} = 0, \tag{116}$$

and (112) simplifies to

$$\frac{\partial H(\cdot)^{III}}{\partial i^{LF}} = 2\phi(\bar{t}^{*III}). \tag{117}$$

Considering (82), applying the implicit function theorem thus yields

$$\frac{\partial L^{*III}}{\partial i^{LF}} = -\frac{\frac{\partial H(\cdot)^{III}}{\partial i^{LF}}}{\frac{\partial H(\cdot)^{III}}{\partial L^{*III}}} = -\frac{2\phi(\bar{t}^{*III})}{\lambda} < 0. \tag{118}$$

For the cross derivative we get

$$\frac{\partial^2 L^{III*III}}{\partial i^{LF}\partial\chi} = -\frac{2(1-c)}{\lambda}\left(\int_{\bar{t}^{*III}}^{t^{max}} t_b g(t_b)\,dt_b - \chi\bar{t}^{*III} g(\bar{t}^{*III})\frac{\partial\bar{t}^{*III}}{\partial\chi}\right) = -\frac{2\phi(\bar{t}^{*III})}{\lambda\chi} < 0. \tag{119}$$

With respect to $\partial\bar{t}^{*III}/\partial\chi = 0$ (see (79)), see analogously our comments on (110). ■

A.7 Proof of Result 5

Equilibrium I

Partially differentiating $H(\cdot)^I$ given by (35) w.r.t. i^{DF} we get

$$\frac{\partial H(\cdot)^I}{\partial i^{DF}} = 0. \tag{120}$$

Considering (37) and (120), and applying the implicit function theorem yields

$$\frac{\partial L^{*I}}{\partial i^{DF}} = -\frac{\frac{\partial H(\cdot)^I}{\partial i^{DF}}}{\frac{\partial H(\cdot)^I}{\partial L^{*I}}} = 0. \tag{121}$$

■

Equilibrium II

It follows from (39) that

$$\frac{\partial H(\cdot)^{II}}{\partial i^{DF}} = -2\gamma(1-c)\chi\bar{t}^{*II} g(\bar{t}^{II*})\frac{\partial\bar{t}^{*II}}{\partial i^{DF}}, \tag{122}$$

$$\text{where} \quad \frac{\partial\bar{t}^{*II}}{\partial i^{DF}} = \frac{\partial\bar{t}^{*II}}{\partial RO^{*II}}\frac{\partial RO^{*II}}{\partial i^{DF}} = \frac{1}{(1-c)\chi L^{*II}}\frac{\partial RO^{*II}}{\partial i^{DF}}, \tag{123}$$

and from (46) that

$$\frac{\partial Z(\cdot)^{II}}{\partial i^{DF}} = \begin{cases} 1 & \text{if} \quad \bar{t}^{*II} > 0, \\ 0 & \text{if} \quad \bar{t}^{*II} < 0. \end{cases} \tag{124}$$

Considering (49), applying the implicit function theorem thus yields

$$\frac{\partial RO^{*II}}{\partial i^{DF}} = -\frac{\frac{\partial Z(\cdot)^{II}}{\partial i^{DF}}}{\frac{\partial Z(\cdot)^{II}}{\partial RO^{*II}}} = \begin{cases} \frac{(1-c)\chi L^{*II}}{2\gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} > 0, \\ 0 & \text{if} \quad \bar{t}^{*II} < 0, \end{cases} \tag{125}$$

which means that (123) becomes

$$\frac{\partial \bar{t}^{*II}}{\partial i^{DF}} = \begin{cases} \frac{1}{2\gamma g(\bar{t}^{*II})} & \text{if} \quad \bar{t}^{*II} > 0, \\ 0 & \text{if} \quad \bar{t}^{*II} < 0, \end{cases} \tag{126}$$

so that (122) simplifies to

$$\frac{\partial H(\cdot)^{II}}{\partial i^{DF}} = \begin{cases} -(1-c)\chi \bar{t}^{*II} & \text{if} \quad \bar{t}^{*II} > 0, \\ 0 & \text{if} \quad \bar{t}^{*II} < 0. \end{cases} \tag{127}$$

Considering (55) and (127), applying the implicit function theorem yields

$$\frac{\partial L^{*II}}{\partial i^{DF}} = -\frac{\frac{\partial H(\cdot)^{II}}{\partial i^{DF}}}{\frac{\partial H(\cdot)^{II}}{\partial L^{*II}}} = \begin{cases} \frac{(1-c)\chi \bar{t}^{*II}}{\lambda} > 0 & \text{if} \quad \bar{t}^{*II} > 0, \\ 0 & \text{if} \quad \bar{t}^{*II} < 0, \end{cases} \tag{128}$$

which leads to

$$\frac{\partial^2 L^{*II}}{\partial i^{DF} \partial \chi} = \begin{cases} \frac{1-c}{\lambda}\left[\bar{t}^{*II} + \chi \frac{\partial \bar{t}^{*II}}{\partial \chi}\right] = \frac{(1-c)\bar{t}^{*II}}{\lambda} > 0 & \text{if} \quad \bar{t}^{*II} > 0, \\ 0 & \text{if} \quad \bar{t}^{*II} < 0. \end{cases} \tag{129}$$

$$\frac{\partial^2 L^{*II}}{\partial i^{DF} \partial \gamma} = \begin{cases} \frac{(1-c)\chi}{\lambda} \frac{\partial \bar{t}^{*II}}{\partial \gamma} = \frac{(1-c)\chi(1-G(\bar{t}^{*II}))}{\lambda \gamma g(\bar{t}^{*II})} > 0 & \text{if} \quad \bar{t}^{*II} > 0, \\ 0 & \text{if} \quad \bar{t}^{*II} < 0. \end{cases} \tag{130}$$

With respect to $\partial \bar{t}^{*II}/\partial \chi$ and $\partial \bar{t}^{*II}/\partial \gamma$ see analogously our comments on (110) and (111).

■

Equilibrium III

It follows from (66) that

$$\frac{\partial H(\cdot)^{III}}{\partial i^{DF}} = (1-c)\chi \left[-\int_{\bar{t}^{*III}}^{t^{max}} t_b g(t_b)\, dt_b - (i^{LF} - i^{DF})\bar{t}^{*III} g(\bar{t}^{*III}) \frac{\partial \bar{t}^{*III}}{\partial i^{DF}} \right], \tag{131}$$

$$\text{where} \quad \frac{\partial \bar{t}^{*III}}{\partial i^{DF}} = \frac{\partial \bar{t}^{*III}}{\partial RO^{*III}} \frac{\partial RO^{*III}}{\partial i^{DF}} = \frac{1}{(1-c)\chi L^{*III}} \frac{\partial RO^{*III}}{\partial i^{DF}}, \tag{132}$$

and from (73) that

$$\frac{\partial Z(\cdot)^{III}}{\partial i^{DF}} = G(\bar{t}^{*III}). \tag{133}$$

Considering (76), applying the implicit function theorem thus yields

$$\frac{\partial RO^{*III}}{\partial i^{DF}} = -\frac{\frac{\partial Z(\cdot)^{III}}{\partial i^{LF}}}{\frac{\partial Z(\cdot)^{III}}{\partial RO^{*III}}} = \frac{G(\bar{t}^{*III}(1-c)\chi L^{*III}}{(i^{LF} - i^{DF}) g(\bar{t}^{*III})}, \tag{134}$$

which means that (132) becomes

$$\frac{\partial \bar{t}^{*III}}{\partial i^{DF}} = \frac{G(\bar{t}^{*III})}{(i^{LF} - i^{DF}) g(\bar{t}^{*III})}, \tag{135}$$

so that (131) simplifies to

$$\frac{\partial H(\cdot)^{III}}{\partial i^{DF}} = -\phi(\bar{t}^{*III}) - (1-c)\chi \bar{t}^{*III} G(\bar{t}^{*III}). \tag{136}$$

Considering (82) and (136), applying the implicit function theorem yields

$$\begin{aligned} \frac{\partial L^{*III}}{\partial i^{DF}} = -\frac{\frac{\partial H(\cdot)^{III}}{\partial i^{DF}}}{\frac{\partial H(\cdot)^{III}}{\partial L^{*III}}} &= \frac{(1-c)\chi \left[\int_{\bar{t}^{*III}}^{t^{max}} t_b g(t_b)\, dt_b + \bar{t}^{*III} G(\bar{t}^{*III}) \right]}{\lambda} \\ &= \frac{\phi(\bar{t}^{*III}) + (1-c)\chi \bar{t}^{*III} G(\bar{t}^{*III})}{\lambda} > 0, \end{aligned} \tag{137}$$

so that

$$\frac{\partial^2 L^{*III}}{\partial i^{DF}\partial\chi} = \frac{(1-c)}{\lambda}\left[\left(\int_{\bar{t}^{*III}}^{t^{max}} t_b g(t_b)\,dt_b + \bar{t}^{*III}G(\bar{t}^{*III})\right)\right.$$
$$\left.+\chi\left(-\bar{t}^{*III}g(\bar{t}^{*III})\frac{\partial\bar{t}^{*III}}{\partial\chi} + \frac{\partial\bar{t}^{*III}}{\partial\chi}G(\bar{t}^{*III}) - \bar{t}^{*III}g(\bar{t}^{*III})\frac{\partial\bar{t}^{*III}}{\partial\chi}\right)\right]$$
$$= \frac{\phi(\bar{t}^{*III}) + (1-c)\chi\bar{t}^{*III}G(\bar{t}^{*III})}{\lambda\chi} > 0. \tag{138}$$

With respect to $\partial\bar{t}^{*III}/\partial\chi = 0$ (see (79)), see analogously our comments on (110). Note that the term $\int_{\bar{t}^{*III}}^{t^{max}} t_b g(t_b)\,dt_b + \bar{t}^{*III}G(\bar{t}^{*III}) > 0$ (see notes to (56)). ■

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