Frictions in the Interbank Market and Uncertain Liquidity Needs: Implications for Monetary Policy Implementation

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Abstract

This paper analyzes in how far frictions in the overnight interbank market influence the impact of monetary policy on bank loan supply and discusses the implications for monetary policy implementation. We find that frictions in the interbank market are not an obstacle for monetary policy to have an impact on bank loan supply. On the contrary, sufficiently high frictions may even amplify the effectiveness of monetary policy. With respect to the implications for monetary policy implementation we show that the corridor which the rates on the central bank’s facilities form around its main refinancing rate is crucial.

JEL classification: E52, E58, G21

Keywords: interbank market, monetary policy, monetary policy implementation, interest corridor, loan supply

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1 Introduction

The interbank market for overnight loans is important for monetary policy implementation. By steering the interest rate in this market, the central bank aims to influence short-term nominal interest rates, and thereby, through various channels, the price level and maybe aggregate output. In the euro area, the interest rate channel and the credit channel play an important role. Both transmission mechanisms rest on the central bank’s ability to influence bank lending. However, during the recent financial crisis, euro area interbank markets seized up. This led to concerns about the Eurosystem’s ability, or the lack thereof, to actually control bank lending in times of malfunctioning interbank markets, and it triggered a heated debate of whether the transmission mechanism of monetary policy might be impaired. Our paper aims to contribute to this debate first, by studying in how far frictions in the interbank market for overnight loans influence the impact of monetary policy on bank loan supply and second, by discussing the implications for monetary policy implementation.

We develop a theoretical model that has two central features. First, it accounts for interbank market frictions. Second, it captures main elements of the Eurosystem’s operational framework. Frictions in the interbank market emerge in the form of transaction costs. We broadly interpret these costs as search costs. Banks must find suitable transaction partners with first, matching liquidity needs and second, a willingness to conclude mutual agreements for trade. The former may be costly as, for example, banks have to split large transactions into small ones to work around credit lines (Bartolini, Bertola, and Prati, 2001). The latter may be costly because lenders in the overnight interbank market are typically unwilling to expose themselves to any counterparty credit risk (Hauck and Neyer, 2014). Consequently, they engage in costly checks of the creditworthiness of potential borrowers who in turn must provide costly signals of their creditworthiness.

1 For respective empirical analyses for the interest rate channel see, for example, Čihák, Harjes, and Stavrev (2009) and Angeloni, Kashyap, Mojon, and Terlizzese (2003). In an empirical analysis referring to the Spanish credit market, Jiménez, Ongena, Peydró, and Saurina (2012) confirm a high relevance of the credit channel.

2 The term “Eurosystem” stands for the institution which is responsible for monetary policy in the euro area, namely the ECB and the national central banks in the euro area. For the sake of simplicity, the terms “ECB” and “Eurosystem” are used interchangeably throughout this paper.

3 Bartolini, Bertola, and Prati (2001) are among the first dealing explicitly with interbank market transaction costs. They argue that interbank market transaction costs are responsible for the relatively high federal funds rate usually observed at the end of a reserve maintenance period. Transaction costs also
Main elements of the Eurosystem’s operational framework captured by our model are the main refinancing operations and the two standing facilities. The main refinancing operations are credit operations with a maturity of one week by which the Eurosystem provides reserves to the banking sector. The two standing facilities, a deposit facility and a lending facility, allow banks to balance their overnight liquidity needs. The interest rates on the facilities form a corridor around the rate on the main refinancing operations with the deposit rate to be lower and the lending rate to be higher than the main refinancing rate.\(^4\) Our model results replicate stylized facts observed during the recent financial crisis: an extremely low interbank rate, being only slightly above the deposit rate, and a significantly increased demand for central bank liquidity outweighing by far the banking sector’s structural liquidity needs\(^5\) implying a massive use of the central bank’s deposit facility. Although our model focuses on the Eurosystem’s operational framework, our results also apply to other operational frameworks which allow commercial banks to balance uncertain liquidity needs by using a deposit facility and a lending facility offered by the central bank.

With respect to the above outlined aim of this paper, we find that frictions in the interbank market for overnight loans are not an obstacle for monetary policy to have an impact on bank loan supply. On the contrary, sufficiently high frictions may even amplify the effectiveness of monetary policy. The resulting implications for monetary policy implementation can be summarized as follows:

- If the central bank changes all its interest rates (main refinancing rate and the rates on its two facilities) to the same extent, so that the interest corridor remains constant, only banks’ certain marginal funding costs will change. A decline in all interest rates implies certain marginal funding costs to decrease resulting in an increase in bank

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\(^4\)For a detailed description of the Eurosystem’s operational framework see European Central Bank (2012a).

\(^5\)Generally, the banking sector’s structural liquidity needs correspond to the Eurosystem’s benchmark allotment. This is the allotment in the main refinancing operations that will allow banks to smoothly fulfil their reserve requirements taking into account future liquidity needs from reserve requirements and autonomous factors. For details see European Central Bank (2014a).
loan supply. The effectiveness of this monetary policy impulse is independent of interbank market frictions. For its impact on bank loan supply the sign of the deposit rate is irrelevant.

• If, in contrast, the central bank changes the interest corridor by altering its width or asymmetry, bank’s uncertain marginal funding costs will change, provided interbank market frictions are sufficiently high. Narrowing the width of the corridor results in lower uncertain marginal funding costs imposing an expansionary effect on bank loan supply. Implementing an asymmetric corridor or changing the asymmetry of the corridor by, for example, decreasing only the main refinancing rate, results in a decrease in both, certain and uncertain marginal costs. Compared with the case in which the central bank changes all interest rates to the same extent, the expansionary effect on bank loan supply is thus stronger. The effectiveness of these monetary policy measures increase in the extent of interbank market frictions. Again, the sign of the deposit rate plays no role for the impact on bank loan supply.

The main idea of our model and its results can be described as follows. The banking sector faces a certain structural liquidity deficit that can only be covered by borrowing from the central bank. Due to uncertain deposit transfers by its customers each bank additionally faces uncertain liquidity needs. As the deposit outflow of a bank implies a respective deposit inflow of another bank, banks may exclusively use the interbank market to balance these liquidity needs. Frictions in this market in the form of transaction costs are crucial as they may influence the effectiveness of monetary policy.

For low transaction costs banks cover only their certain structural liquidity deficit by borrowing from the central bank’s refinancing operations and balance their uncertain liquidity needs exclusively via the interbank market. If the central bank decreases the main refinancing rate, certain marginal funding costs will decrease leading to an expansionary effect on bank loan supply. Uncertain marginal funding costs will remain unchanged. If transaction costs are that large that they exceed a specific threshold, banks will have a precautionary demand for liquidity. They borrow more liquidity from the central bank than they need to cover their structural liquidity deficit. The resulting excess liquidity has to be placed in the deposit facility. The spread between the main refinancing rate and the
deposit rate determines expected marginal costs and revenues of holding precautionary liquidity. If the central bank decreases this spread, precautionary liquidity will become cheaper so that banks will increase their holdings of this liquidity. This bank behavior reduces their uncertain marginal funding costs resulting in an expansionary impulse on bank loan supply. If transaction costs become so high that they exceed a further threshold, the interbank market will break down. In this case, banks do not only have a precautionary demand for liquidity, but they also have to use either the deposit facility or the lending facility to balance their uncertain liquidity needs. In this case, an increase in the deposit rate and/or a decrease in the lending rate result in lower uncertain marginal funding costs so that bank loan supply increases. Consequently, our model shows that for sufficiently high transaction costs the interest corridor can be used as an effective monetary policy instrument as it is a determinant of banks’ uncertain marginal funding costs.

The rest of this paper is organized as follows. Section 2 presents related literature. Section 3 describes the framework of the model. Sections 4 derives the optimal behavior of an individual bank while Section 5 considers the aggregate level. Section 6 discusses the equilibrium of the model. Taking a closer look at this equilibrium in Section 7, we analyze the impact of monetary policy on bank loan supply and discuss the consequences for monetary policy implementation. Section 8 concludes the paper.

2 Related Literature

Our paper contributes to three strands of literature. The first strand focuses on the influence of monetary policy on bank lending. A huge part of this literature considers asymmetric information in credit markets and argues that these frictions amplify the effects of monetary policy on bank lending and, therefore, on aggregate demand. This so called credit view of monetary policy can be divided into the balance sheet channel and the bank lending channel. With respect to the balance sheet channel, the crucial point is that a monetary policy impulse changes borrowers’ net worth. A contractionary monetary policy decreases borrowers’ net worth which implies an increase in adverse selection and moral hazard problems leading to a decline in bank lending. Seminal papers are those by

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6For a survey see, for example, Boivin, Kiley, and Mishkin (2010) and Peek and Rosengreen (2012).
Bernanke and Gertler (1989) and Bernanke, Gertler, and Gilchrist (1999). With respect to the bank lending channel, the driving force is that monetary policy has an impact on bank deposits. A contractionary monetary policy reduces bank deposits implying a decline in bank loan supply. Important papers dealing with this traditional bank lending channel are, for example, those by Gertler and Gilchrist (1993) and Kashyap and Stein (1995). This traditional approach of the bank lending channel has been criticized as it neglects, for example, that banks can replace deposits by market-based funding. However, Disyatat (2011) shows that a greater reliance on market-based funding creates a new approach of the bank lending channel. Crucial is that a stronger reliance on market-based funding increases the sensitivity of banks’ funding costs to monetary policy as banks’ health, in terms of leverage, asset quality and in perception of risk, becomes more important.

The second strand of literature deals with frictions in the interbank market. Until the outbreak of the financial crisis in 2007, the interbank market was typically regarded as frictionless. As a consequence, the interbank rate was assumed to be identical to the monetary policy rate or the interbank market was entirely neglected. The financial crisis challenged this view and inspired a growing literature dealing with interbank market imperfections, primarily focusing on asymmetric information about credit risks. Freixas and Jorge (2008) consider the impact of this friction for the transmission of monetary policy. They show that private information with respect to credit risks may induce rationing of firms in credit markets. With respect to the transmission mechanism of monetary policy this implies that asymmetric information in the interbank market may be responsible for a) a magnitude effect, i.e. the aggregate impact of monetary policy may be large given the small interest elasticity of investment, and b) a liquidity effect, i.e. that the impact of monetary policy is stronger for banks with less liquid balance sheets. Heider, Hoerova, and Holthausen (2009) argue that banks’ informational disadvantage with respect to counterparty credit risks induces them to hold more liquidity. Depending on the risk dispersion, this may result in either adverse selection or a dry-up in the interbank market. However, banks may learn about counterparty credit risks by repeatedly trading with each other so that the asymmetric information problem may be mitigated. In an empirical analysis of the German unsecured overnight money market, Bräuning and Fecht (2012) determine
the impact of such relationship lending for banks’ ability to access liquidity. Focusing on the impact of the interbank market break down during the crisis on Portuguese bank loan supply, Iyer, Peydró, da Rocha-Lopes, and Schoar (2014) identify that the overall contractionary effect is intensified for those banks which have less relationship lending, are more active in the interbank market and are smaller in size. The causes of a possible dry-up in the interbank market are also analyzed by Allen, Carletti, and Gale (2009). They show that banks will start to hoard liquidity if they are unable to hedge idiosyncratic liquidity shocks.

The third strand of literature deals with monetary policy implementation, bank behavior, and consequences for the conditions in the overnight interbank market. This literature can be divided into three groups. The first group focuses on the U.S. before the outbreak of the financial crisis in 2007. Considering major institutional characteristics of the federal funds market, Ho and Saunders (1985) as well as Clouse and Dow (2002) analyze banks’ reserve management and draw conclusions for the conditions in the interbank market for reserves. However, the largest part of the literature dealing with the federal funds market focuses on why the federal funds rate fails to follow a martingale within the reserve maintenance period.7

The second group refers to the euro area in the pre-crisis period. A bulk of this literature deals with the under- and overbidding behavior in the Eurosystem’s main refinancing operations which could be observed in the first years of the European Monetary Union.8 Apart from this, there are papers analyzing the consequences of alternative monetary policy implementations. Nautz (1998) shows that the central bank can influence the interbank market rate by being more or less vague about its future monetary policy. Välimäki (2001) analyzes the effects of alternative tender procedures with respect to the Eurosystem’s refinancing operations. Neyer and Wiemers (2004) refer to the collateral framework. They show that differences in banks’ opportunity costs of holding collateral form a rationale for the existence of an interbank market for reserves. Neyer (2009)

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8Under- and overbidding behavior refers to a bidding behavior in which total bids significantly deviate from the Eurosystem’s benchmark allotment. Analyses with respect to this under- and overbidding behavior can be found in e.g. Ayuso and Repullo (2001, 2003), Ewerhart (2002), Nautz and Oechssler (2003, 2006), and Bindseil (2005).
demonstrates that remunerating required reserves in a specific way increases the flexibility of monetary policy. Pérez-Quirós and Rodríguez-Méndizábal (2006) show that the two standing facilities offered by the ECB in combination with its minimum reserve system are an effective instrument to stabilize the interbank rate. Whitesell (2006), although not explicitly referring to the euro area, looks at a minimum reserve system and standing facilities as two alternative regimes for controlling overnight interest rates. Also focusing on the standing facilities, Berentsen and Monnet (2008) develop a general equilibrium framework and show that changing the rates on these facilities may be used actively as a monetary policy instrument. Beaupain and Durré (2008) examine the interday and intraday dynamics of the euro area overnight interbank market and argue that specific features of the ECB’s operational framework, as its minimum reserve system, can explain observed regular patterns.

The third group of this third strand of literature comprises papers dealing with monetary policy implementation during the financial crisis.9 Especially in response to occurring tensions in interbank markets, central banks adopted several non-standard-measures. Borio and Disyatat (2009) point out that an important feature of these policies is that the central bank also uses its balance sheet to influence prices and conditions in the interbank market. Cheun, von Köppen-Mertes, and Weller (2009) consider changes to the collateral frameworks of the Eurosystem, the Federal Reserve System and the Bank of England. Lenza, Pill, and Reichlin (2010) describe the way in which these three central banks generally conducted monetary policy during the financial crisis and point to the importance of their influence on money market spreads. Eisenschmidt, Hirsch, and Linzert (2009) analyze the relatively aggressive bidding behavior of banks in the ECB’s main refinancing operations at the beginning of the financial turmoil. Also referring to the first part of the financial crisis (until 2008), Cassola and Huetl (2010) assess the effectiveness of monetary policy implementation during that time. Hauck and Neyer (2014) develop a theoretical model considering main institutional features of the ECB’s operational framework which has been in place since September 2008 to explain several stylized facts observed during the financial crisis.

Our paper combines all three strands of this literature by analyzing the consequences of frictions in the overnight interbank market, in the form of broadly defined transaction costs, for the impact of monetary policy on bank loan supply. With respect to monetary policy implementation, we point out that the central bank's standing facilities play a crucial role for the effectiveness of monetary policy in the presence of interbank market frictions and uncertain liquidity needs.

3 Setup

In our model economy we consider a continuum of measure one of price-taking commercial banks with a large number of bank customers and a central bank. Each commercial bank $b$ grants a loan volume $L_b$ to its customers and credits the amount to their demand deposit accounts. Bank customers can use this newly created money to make payments. They pay by cash or by transferring deposits. Each bank $b$ experiences cash withdrawals $cL_b$, with the currency ratio $c$ reflecting the share of the newly created money used for cash payments. The share $\chi t_b$ of the remaining deposits $(1-c)L_b$ is transferred to customers of other banks.

Crucial is that the net deposit transfer differs across banks. This is reflected by the bank individual deposit transfer variable $t_b$. For banks facing a net deposit inflow $t_b < 0$, and for those facing a net deposit outflow $t_b > 0$. For a single bank $b$, its net deposit transfer is uncertain as $t_b$ is the realization of a random variable $T$. Across all banks, $t_b$ is distributed in the interval $[t_{\text{min}}, t_{\text{max}}]$ according to the density function $g(t_b) = G'(t_b)$ with

$$E[T] = \int_{t_{\text{min}}}^{t_{\text{max}}} t_b g(t_b) \, dt_b = 0. \quad (1)$$

The parameter $\chi$ with $\chi \in (0, \frac{1}{t_{\text{max}}}]$ is a scale parameter determining the dispersion of the distribution of $T$. As the share $\chi t_b$ cannot exceed one, $\chi$ is restricted to $\frac{1}{t_{\text{max}}}$. If $\chi$ increases, the distribution will exhibit a higher dispersion. Accordingly, we use $\chi$ as a measure for uncertainty about each bank’s net deposit transfer.

\footnote{As the share $\chi t_b$ cannot exceed one, $\chi$ is restricted to $\frac{1}{t_{\text{max}}}$.}
Bank $b$ can balance its individual liquidity needs by transacting with the central bank or in the interbank market. However, cash withdrawals imply that the banking sector as a whole faces a structural liquidity deficit that can only be covered by the central bank being the monopoly producer of currency.

At the time when bank $b$ decides on its optimal loan supply $L_b$, it can obtain liquidity by borrowing $RO_b$ from the central bank’s main refinancing operations at the rate $i^{RO}$, henceforth, policy rate. When making these decisions, the bank does not know the realization $t_b$ of the random variable $T$ yet. The bank thus faces uncertainty about its future liquidity needs.

This uncertainty is resolved once bank customers withdrew cash and made transfer payments. Then, bank $b$’s liquidity needs are given by

$$N_b := (c + (1 - c)\chi t_b) L_b - RO_b \geq 0. \quad (2)$$

From (2) we can infer that bank $b$ will face neither a liquidity deficit nor surplus ($N_b = 0$) if

$$t_b = \frac{RO_b - c L_b}{(1-c)\chi L_b} =: \bar{t}_b. \quad (3)$$

Consequently, bank $b$ will face a liquidity deficit $N_b > 0$ if $t_b > \bar{t}_b$ and a surplus $N_b < 0$ if $t_b < \bar{t}_b$. As shown by (3), the critical deposit transfer variable $\bar{t}_b$ will decrease if $RO_b$ becomes smaller. Due to the non-negativity constraint on $RO_b$, it thus follows that

$$\bar{t}_b \geq -\frac{c}{(1-c)\chi} =: \bar{t}. \quad (4)$$

To balance its liquidity needs $N_b$ the bank decides on its transactions in the interbank market $B_b$ as well as on its use of the central bank’s facilities. It may borrow $LF_b$ at the rate $i^{LF}$ from the lending facility and place $DF_b$ at the rate $i^{DF}$ in the deposit facility.\footnote{Generally, credit operations with the central bank require adequate collateral. In our setting a bank’s loan volume $L_b$ serves as collateral, and therefore, limits its central bank borrowing. The central bank may impose a haircut on these loans when accepting them as collateral, like in Bindseil and König (2011). In this setting, however, we assume that such a haircut is not binding and neglect the collateralization of central bank loans. See in this context also our remarks on this aspect made in Section 7.5.} Borrowing from the refinancing operation is no longer feasible.
facilities form a corridor around the rate on the refinancing operations $i^{RO}$ with $i^{LF} > i^{RO} > i^{DF}$.

This sequence of moves implies that the optimization problem can be split up into two stages. At the second stage, bank $b$ aims to minimize its liquidity costs $C_b$ resulting from using the interbank market and the facilities. The optimization problem reads

$$\min_{B_b, LF_b, DF_b} C_b(B_b, LF_b, DF_b) = i^{IBM} B_b + \gamma |B_b| + i^{LF} LF_b - i^{DF} DF_b$$

(5)

subject to

$$N_b = B_b + LF_b - DF_b,$$

(6)

where (6) describes bank $b$'s balance sheet constraint. The first two terms of (5) reflect the bank’s costs or revenues of using the interbank market. Its position in this market is $B_b$. If $B_b > 0$, bank $b$ will borrow the amount $B_b$ at the rate $i^{IBM}$. Conversely, $B_b < 0$ indicates that bank $b$ will lend the amount $|B_b|$ at this rate. In both cases, transaction costs $\gamma |B_b|$ accrue, with $\gamma \geq 0$. The last two terms of (5) reflect bank $b$’s interest costs and revenues from using the central bank’s facilities.

At the first stage, bank $b$ aims to maximize its expected profit $E[\pi_b]$. Indicating the optimum variables of the second stage by the superscript $opt$, the optimization problem at the first stage is given by

$$\max_{L_b, RO_b \in \mathbb{R}^+} E[\pi_b] = i^L L_b - \frac{1}{2} \lambda L_b^2 - i^{RO} RO_b - E \left[ C_b \left( B_b^{opt}, LF_b^{opt}, DF_b^{opt} \right) \right].$$

(7)

Bank $b$ materializes a return by granting loans $L_b$ at a given interest rate $i^L$. Managing these loans is costly, as indicated by the second term of (7). The quadratic form of this cost function captures the idea that loans differ in their complexity so that the bank adds the least complex loans to its portfolio first. Moreover, costs accrue from borrowing liquidity from the central bank’s refinancing operation $RO_b$ at the rate $i^{RO}$. In addition, bank $b$ has to consider the expected liquidity costs $E[C_b]$ from using the interbank market and the facilities.
4 Optimal Behavior of a Single Bank

In order to determine bank b’s optimal liquidity management and its optimal loan supply to the non-banking sector, we solve the optimization problem by backward induction. Accordingly, we first describe briefly bank b’s optimal transactions in the interbank market and its optimal use of the central bank’s facilities at the second stage. Then, we analyze the first stage of the model and determine each bank’s optimal loan supply to the non-banking sector and its optimal borrowing from the central bank’s refinancing operations.

4.1 Optimal Use of the Interbank Market and the Facilities

At the second stage, bank b optimally balances its liquidity needs \( N_b \) for a given interbank rate \( i^{BM} \). If \( N_b > 0 \), bank b will inherit a liquidity deficit. In this case, the bank compares marginal costs of borrowing from the interbank market given by \( i^{BM} + \gamma \) with those of using the lending facility which are simply \( i^{LF} \). As the two marginal costs are constant, the bank will cover its total liquidity deficit by borrowing from the lending facility if \( i^{BM} + \gamma > i^{LF} \). If \( i^{BM} + \gamma < i^{LF} \), it will borrow exclusively from the interbank market. In case both marginal costs are identical, the bank is essentially indifferent between interbank borrowing and the usage of the lending facility.

If \( N_b < 0 \), bank b will inherit a liquidity surplus so that the bank decides analogously. If marginal revenues of placing the excess liquidity in the interbank market \( i^{BM} - \gamma \) are higher (lower) than marginal revenues of using the central bank’s deposit facility \( i^{DF} \), it will place its total surplus in the interbank market (in the deposit facility). In case marginal revenues are identical, the bank is again indifferent.

4.2 Optimal Borrowing from the Refinancing Operations

Solving the optimization problem (7) for \( RO_{b}^{opt} \), we obtain

**Lemma 1:** Suppose that \( i^{BM} \in [i^{RO} - \gamma, i^{RO} + \gamma] \). If \( \tilde{t}_{b}^{opt} \geq \tilde{t} := -\frac{c}{(1-c)\chi}, \) bank b will borrow from the central bank’s refinancing operations according to the following first order condition:

\[
i^{RO} = \max \{ i^{BM} - \gamma, i^{DF} \} G(\tilde{t}_{b}^{opt}) + \min \{ i^{BM} + \gamma, i^{LF} \} \left[ 1 - G(\tilde{t}_{b}^{opt}) \right]. \tag{8}
\]
Proof: See appendix.

Bank $b$ balances marginal costs $i^{RO}$ of borrowing from the central bank’s refinancing operations with expected marginal revenues captured by the right hand side of (8). With probability $G(\tilde{t}_b^{opt})$ bank $b$ will face a liquidity surplus at the second stage. In this case, the bank will place its excess liquidity either in the interbank market to obtain $i^{IBM} - \gamma$ or in the deposit facility at the rate $i^{DF}$ depending on which alternative will be more profitable. The first term on the right hand side thus shows expected marginal revenues in the form of expected interest revenues (less transaction costs). With probability $1 - G(\tilde{t}_b^{opt})$, bank $b$ will face a liquidity deficit at the second stage. In order to balance its liquidity needs, bank $b$ will then borrow either from the interbank market at marginal costs $i^{IBM} + \gamma$ or from the central bank’s lending facility at the rate $i^{LF}$, depending on which option will be cheaper. Consequently, the second term on the right hand side represents expected marginal revenues of borrowing from the refinancing operations in the form of avoided illiquidity costs.

As long as marginal costs are higher than expected marginal revenues, the bank has an incentive to reduce its borrowing from the refinancing operations. Borrowing less from the refinancing operations reduces the probability $G(\tilde{t}_b^{opt})$ of facing a liquidity surplus.\footnote{According to (3), borrowing less from the refinancing operations reduces $\tilde{t}_b^{opt}$ and thus $G(\tilde{t}_b^{opt})$.} As marginal revenues in the case of a liquidity surplus are strictly smaller than those in the case of a liquidity deficit ($\max \{i^{IBM} - \gamma, i^{DF}\} < \min \{i^{IBM} + \gamma, i^{LF}\}$), expected marginal revenues will increase if the bank reduces its borrowing from the refinancing operations.

It will turn out that $RO_b = 0$ and $RO_b \to \infty$ are not possible equilibria. In order to reduce complexity we, therefore, already exclude these cases when presenting Lemma 1 and 2 by supposing that $i^{IBM} \in [i^{RO} - \gamma, i^{RO} + \gamma]$ and $\tilde{t}_b^{opt} \geq \tilde{t}$.\footnote{If the non-negativity constraint on $RO_b$ becomes binding, $RO_b^{opt} = 0$. This will be the case, if $\tilde{t}_b^{opt} < \tilde{t}$ (see also (3). Bank $b$ would also refrain from borrowing from the refinancing operations if borrowing from the interbank market were strictly cheaper. This would be the case if $i^{IBM} + \gamma < i^{RO}$. Moreover, the bank would borrow unlimitedly from the refinancing operations to place its liquidity in the interbank market if the resulting marginal revenues $i^{IBM} - \gamma$ were higher than the marginal costs $i^{RO}$.}

### 4.3 Optimal Loan Supply to the Non-Banking Sector

Solving the optimization problem (7) with respect to $L_b^{opt}$, we obtain
Lemma 2: Suppose that $i^{BM} \in [i^{RO} - \gamma, i^{RO} + \gamma]$. If $\bar{t}_i \geq \bar{t} := -\frac{c}{1-c}\chi$, bank $b$ will supply loans to the non-banking sector according to the following first order condition:

$$i^L = \lambda L_{b}^{opt} + ci^{RO} + (1-c)\chi E[T|t_b < \bar{t}_b^{opt}] G(\bar{t}_b^{opt}) \max \{i^{BM} - \gamma, i^{DF}\}$$

$$+ (1-c)\chi E[T|t_b > \bar{t}_b^{opt}] \left[1 - G(\bar{t}_b^{opt})\right] \min \{i^{BM} + \gamma, i^{LF}\}, \quad (9)$$

with $\bar{t}_b^{opt}$ being implicitly defined by (8).

Proof: See appendix.

Optimal loan supply $L_{b}^{opt}$ requires balancing marginal revenues with expected marginal costs of granting loans. Marginal revenues are equal to the interest rate $i^L$. Expected marginal costs consist of marginal management costs $\lambda L_{b}^{opt}$ and expected marginal funding costs. The latter can be divided into certain and uncertain marginal funding costs.

If the bank increases its loans by one unit, additional certain liquidity needs $c$ will accrue because of cash withdrawals. It follows from Lemma 1 that bank $b$ is indifferent between financing this liquidity need at the first or second stage. Consequently, $ci^{RO}$ reflects certain marginal funding costs due to cash withdrawals. Furthermore, there will be additional uncertain liquidity needs due to deposit transfers. The respective expected additional needs are given by $(1-c)\chi E[T] = 0$. Marginal funding costs or marginal revenues due to deposit transfers will thus only accrue if the materialized deposit transfer differs from the expected one or if there exists a precautionary demand for liquidity. If bank $b$ grants one more unit of loans and if a liquidity surplus materializes, the expected additional liquidity surplus is $(1-c)\chi E[T|t_b < \bar{t}_b^{opt}]$. This surplus, that occurs with probability $G(\bar{t}_b^{opt})$, has to be placed either in the interbank market or in the deposit facility. Analogously, $(1-c)\chi E[T|t_b > \bar{t}_b^{opt}]$ comprises the expected additional liquidity deficit if a liquidity deficit occurs while $1 - G(\bar{t}_b^{opt})$ captures the respective probability. This deficit has to be balanced by borrowing from either the interbank market or the lending facility. The third and fourth term on the right hand side of (9) thus reflect uncertain marginal funding costs due to deposit transfers.

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$^{14}$E[T|t_b < \bar{t}_b^{opt}] and E[T|t_b > \bar{t}_b^{opt}] are the means of a respectively truncated distribution of $T$. 

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5 Aggregate Level

After having clarified the optimal behavior of an individual bank, we are now in a position to look at the aggregate level. At the second stage, each bank will only trade liquidity in the interbank market if this is more beneficial than using the respective central bank’s facility. All variables and parameters determining this decision are the same for all banks. Consequently, if \( i^{BM} - \gamma < i^{DF} \), all banks with a liquidity surplus will place their excess liquidity in the deposit facility, and if \( i^{BM} + \gamma > i^{LF} \), all banks with a liquidity deficit will borrow the missing liquidity from the lending facility. This reveals the importance of transaction costs. If

\[
\gamma > \frac{i^{LF} - i^{DF}}{2} =: \bar{\gamma},
\]

transaction costs will be so high that the interbank market will break down. All banks will prefer to use the central bank’s facilities instead.

Only if \( \gamma \leq \bar{\gamma} \), there will be an active interbank market with the interbank rate being determined by the aggregate liquidity position of the banking sector. Denoting aggregate borrowing from the refinancing operations by \( RO \) and aggregate loan supply to the non-banking sector by \( L \), an aggregate liquidity deficit will arise if banks’ cash withdrawals \( cL \) are larger than the aggregate amount obtained in the refinancing operations \( RO \). In this case, competition for scarce liquidity brings the interbank rate to its upper limit \( i^{LF} - \gamma \). A higher interest rate would not be accepted by liquidity deficit banks, as they would prefer to borrow from the central bank’s lending facility instead. If an aggregate liquidity surplus occurs, because cash withdrawals are lower than the aggregate amount of liquidity obtained in the refinancing operations, competition for limited lending possibilities in the interbank market will bring the interbank rate to its lower limit \( i^{DF} + \gamma \). If there is neither an aggregate liquidity deficit nor surplus, neither market side will possess market power. In consequence, any rate within the lower and the upper limit will depict a possible equilibrium.

When deciding on \( L_b \) and \( RO_b \), all banks face the same optimization problem. All banks base their expectations on their individual deposit transfers on the same distribution
function $G(t_b)$. This implies that for any given loan volume $L_b$ and any amount $RO_b$ borrowed from the refinancing operations, each bank has the same expectations about its subsequent liquidity needs. Moreover, uncertainty exists only at the individual level. At the aggregate level, there is no uncertainty. Aggregate deposit transfers must be zero. These two aspects imply that for any $L_b$ and any $RO_b$ a bank knows the aggregate liquidity position of the banking sector and, therefore, the interbank rate (which is only determined by the aggregate liquidity position as argued above).\(^{15}\) As for any $L_b$ and $RO_b$, a bank knows which interbank rate will prevail at the second stage and as for any $L_b$ and $RO_b$, all banks form the same expectations about their liquidity needs, all banks face exactly the same decision problem at the first stage. This implies that optimal individual borrowing from the refinancing operations $RO_b^{opt}$, the optimal individual loan supply to the non-banking sector $L_b^{opt}$, and therefore also the critical deposit transfer variable $t_b^{opt}$ are identical for all banks. As we have a continuum of banks of unit mass, these bank-individual optimal values correspond to the respective aggregate variables $L$ and $RO$. Furthermore, we can use $\bar{t}$ as the for all banks identical optimal critical deposit transfer variable.

6 **Equilibrium**

In the euro area, aggregate borrowing from the ECB’s main refinancing operations has been systematically equal to or higher than the ECB’s benchmark allotment. As in our model aggregate cash withdrawals $cL$ correspond to the ECB’s benchmark allotment,\(^{16}\) we focus in our analysis on equilibria in which $RO \geq cL$. These equilibria will emerge if there is a left-skewed distribution of $T$ so that $G(0) < 0.5$ and/or if the interest corridor is characterized by $i^{LF} - i^{RO} \geq i^{RO} - i^{DF}$. However, for the sake of completeness we briefly comment on the consequences of a right-skewed distribution and a different asymmetry of the interest corridor at the end of this section.

\(^{15}\)For a similar approach with respect to the interbank rate see, for example, Bartolini, Bertola, and Prati (2001).

\(^{16}\)See Footnote 5 for a description of the benchmark allotment. Note that in our model there are no reserve requirements and autonomous factors consist of currency holdings only. Consequently, in our model aggregate cash withdrawals $cL$ correspond to the ECB’s benchmark allotment.
The assumed symmetry/asymmetry of the interest corridor corresponds to the so far observed cases in the euro area. The left-skewed distribution of $T$ implies for each single bank that the probability of facing a net deposit outflow is larger than the probability of facing a net deposit inflow. Due to $E[T] = 0$, each individual bank thus expects small outflows with a high probability and large inflows with low probability. To motivate the left-skewed distribution, assume that banks have two different types of bank customers. First, each bank possesses a huge number of bank customers predominantly generating relatively small deposit outflows, such as households making payments for consumption purposes. Second, each bank has a small number of bank customers predominantly receiving relatively large payments, such as firms as suppliers of consumption goods. With a small probability one of these firms may benefit from sudden spikes in demand for its goods. In consequence, their respective bank would face a large net deposit inflow. Moreover, a large net deposit inflow may also occur if outflows decline to a large extent. This will be the case if, for example, in a region cash card payments are not feasible due to a power failure. As the probability of such a shock is also small, banks expect a large net deposit inflow with a small probability and small net deposit outflows with a large probability.

Denoting equilibrium variables by an asterisk and combining the considerations on the interbank rate made in Section 5 with the results of Lemma 1 and 2, we obtain

**Proposition 1:** Assume that $G(0) < 0.5$ and that $i^L - i^{RO} > i^{RO} - i^{DF}$. Then, depending on $\gamma$, we have to distinguish between three equilibria $j = \{I, II, III\}$:

$I$: $RO^\ast = cL^\ast$, $DF^\ast = 0$, $LF^\ast = 0$, $i^{IBM^\ast} = i^{RO} - \gamma [1 - 2G(0)]$ if $\gamma \leq \bar{\gamma}$, (11)

$II$: $RO^\ast > cL^\ast$, $DF^\ast > 0$, $LF^\ast = 0$, $i^{IBM^\ast} = i^{DF} + \gamma$ if $\gamma \in (\bar{\gamma}, \bar{\bar{\gamma}}]$, (12)

$III$: $RO^\ast > cL^\ast$, $DF^\ast > LF^\ast > 0$ if $\gamma > \bar{\bar{\gamma}}$, (13)

with

$L^\ast = \begin{cases} \frac{1}{\lambda} \left[ i^L - c^{RO} - \left( (1 - c)E[T|b > \bar{T}] \left[ 1 - G(\bar{T}) \right] \right) \right] 2\gamma & \text{if } \gamma \leq \bar{\gamma}, \\
\frac{1}{\lambda} \left[ i^L - c^{RO} - \left( (1 - c)E[T|b > \bar{T}] \left[ 1 - G(\bar{T}) \right] \right) \right] i^{LF} - i^{DF} & \text{if } \gamma > \bar{\bar{\gamma}}, \end{cases}$ (14)

\(^{17}\)From April 1999 until November 2013 the rates on the Eurosystem’s facilities have formed a symmetric corridor around the rate on the main refinancing operations. However, in order to avoid a negative deposit rate, this rate was not decreased in November 2013 contrary to the lending rate and the main refinancing rate. Consequently, there has been an asymmetric interest corridor until June 2014, when the ECB decided to set a negative deposit rate and thereby restored the symmetry of the corridor.
whereas \( \bar{\gamma} := \frac{i^{RO-DF}}{2[1-G(0)]} \), \( \bar{\gamma} \) is defined in (10) and \( T^* \) is implicitly given by (8).

**Proof:** See appendix.

Proposition 1 shows that for aggregate borrowing from the central banks’ refinancing operations and aggregate loan supply to the non-banking sector interbank market transaction costs play a crucial role. For relatively low transaction costs the banking sector covers only its structural liquidity needs by borrowing from the central banks’ refinancing operations. If the transaction costs parameter \( \gamma \) exceeds the threshold \( \bar{\gamma} \), banks start to borrow more liquidity from the refinancing operations than they need to cover the structural liquidity deficit \( R^{O*} > cL^* \), and therefore, than their expected liquidity needs.\(^{18}\)

This means that banks start to hold precautionary liquidity. The resulting excess liquidity in the banking sector has to be placed in the deposit facility. If \( \gamma \) even exceed the threshold \( \bar{\gamma} \), the interbank market will break down. As a consequence banks will further increase their precautionary holdings of liquidity and in addition to the deposit facility, they will also use the lending facility. The amount banks hold as precautionary liquidity determines their uncertain marginal funding costs and therefore, their loan supply to the non-banking sector. In the following, we will comment on the three equilibria in more detail.

**Equilibrium I**

In Equilibrium I, transaction costs are so low that each bank borrows exactly an amount equal to its cash withdrawals from the central bank’s refinancing operations, i.e. \( R^{O*} = cL^* \). The critical deposit transfer is thus \( T^* = 0 \). Liquidity needs resulting from the deposit transfers are solely balanced by using the interbank market. None of the facilities is used. Only this bank behavior implies that the optimality condition given by (8) is fulfilled.

To see this, suppose as a starting point that \( \gamma = 0 \). If banks borrowed an amount larger than their cash withdrawals from the refinancing operations, there would be an aggregate surplus at the second stage bringing the interbank market to its lower bound, which is \( i^{DF} \) for \( \gamma = 0 \). However, this cannot be an equilibrium, as then marginal costs of borrowing from the refinancing operations \( (i^{RO}) \) exceed expected marginal revenues which in this case equal \( i^{DF} \) as shown by the right hand side of (8). Consequently,

\(^{18}\)A bank’s expected liquidity needs due to the cash withdrawals and deposit transfers are \( cL^* + (1 - c)\chi E[T]L_b = cL_b \) as \( E[T] = 0 \).
banks have an incentive to reduce their borrowing from the refinancing operations to balance marginal costs and expected marginal revenues. Analogously, if banks borrowed an amount lower than their cash withdrawals, there would be an aggregate liquidity deficit bringing the interbank rate to its upper bound, which is \( i^{LF} \) for \( \gamma = 0 \). However, this is not an equilibrium either as then, expected marginal revenues of borrowing from the refinancing operations \( i^{LF} \) exceed marginal costs \( i^{RO} \) so that banks wish to increase their borrowing. Accordingly, for \( \gamma = 0 \) an equilibrium will be reached if each bank borrows an amount from the refinancing operations which is equal to its cash withdrawals. This implies that the interbank rate \( i^{IBM*} \) equals the central bank’s policy rate \( i^{RO} \), and, as revealed by (14), bank loan supply depends only on certain marginal funding costs given by \( c^{RO} \), whereas for \( \gamma = 0 \) uncertain marginal funding costs have no impact on bank loan supply.

Consider next that \( \gamma \) becomes positive. Then, expected marginal revenues of borrowing from the central bank’s refinancing operations increase,\(^{19}\) so that each bank has an incentive to increase its borrowing from these operations above \( cL_b \). However, such an aggregate borrowing behavior results in aggregate excess liquidity so that the interbank rate declines. This decline reduces expected marginal revenues of borrowing from the refinancing operations again. Accordingly, the incentive to borrow more than \( cL_b \) becomes weaker. No bank will borrow more than an amount equal to its cash withdrawals from the refinancing operations if the interbank rate decreases to \( i^{IBM} = i^{RO} - \frac{1}{2} \left[ 1 - 2G(t^* = 0) \right] \). Consequently, as long as the interbank rate is unrestricted, an increase in expected marginal revenues due to higher transaction costs will thus be offset by a decrease in the interbank rate. Banks have no incentive to borrow more than an amount equal to their cash withdrawals from the central bank’s refinancing operations. Equilibrium I as described by (11) will be realized. As shown by (14), for \( \gamma > 0 \) bank loan supply \( L^* \) does not only depend on certain but also on uncertain marginal funding costs. These costs increase in both, the extent of uncertainty \( \chi \) and transaction costs parameter \( \gamma \): If \( \gamma \) increases, a liquidity deficit will

\(^{19}\)If a bank faces a liquidity surplus at the second stage, its marginal revenues from placing liquidity in the interbank market will decrease but its avoided marginal illiquidity costs in the case of a liquidity deficit will increase (see Lemma 1). Due to the left-skewed distribution of \( T \), at \( T = 0 \) the probability of facing a liquidity deficit at the second stage is higher than the probability of facing a liquidity surplus, i.e. for \( RO = cL \). Consequently, marginal costs in the case of a deficit have a higher weight than marginal revenues in the case of a surplus.
be more expansive and a liquidity surplus will generate a lower return. Moreover, if $\chi$ increases, the expected liquidity surplus $(1 - c)\chi E[T|t_b < t^*]$ and the expected liquidity deficit $(1 - c)\chi E[T|t_b > t^*]$ will increase. In consequence, bank loan supply declines in both, transaction costs and the extent of uncertainty.

**Equilibrium II**

If $\gamma$ reaches the threshold $\bar{\gamma}$, the interbank rate will be at its lower bound $i^{DF} + \gamma$. Further decreases in the interbank rate will not be possible to balance marginal costs and expected marginal revenues of borrowing from the refinancing operations. Expected marginal revenues will thus exceed marginal costs if $\gamma$ increases. Accordingly, banks start to increase their borrowing from the refinancing operations reducing the probability of facing a liquidity deficit and, therefore, expected marginal revenues. Each bank borrows more liquidity from the refinancing operations than it expects to need, i.e. a precautionary demand for liquidity emerges. An aggregate liquidity surplus $RO^* > cL^*$ accrues. This excess liquidity can only be placed in the deposit facility. However, all banks with a liquidity deficit are still able to cover their liquidity needs by using the interbank market. Equilibrium II given by (12) materializes.

Rewriting (8) we get for the first order condition for banks’ optimal precautionary liquidity holdings

$$i^{RO} - i^{DF} G(t^*) = (i^{DF} + 2\gamma) [1 - G(t^*)].$$

(15)

The left hand side of (15) represents expected marginal costs of holding precautionary liquidity. Banks borrow this liquidity at the rate $i^{RO}$. With probability $G(t^*)$ they do not need this liquidity and have to place it for a lower return $i^{DF}$ in the interbank market or in the deposit facility both yielding a (net) return of $i^{DF}$. The right hand side of (15) shows expected marginal revenues of holding precautionary liquidity. With probability $1 - G(t^*)$ each bank faces a liquidity deficit. Banks have to cover this deficit by borrowing from the interbank market at the cost of $i^{IBM*} + \gamma = i^{DF} + 2\gamma$. The right hand side thus reflects expected marginal revenues of holding precautionary liquidity in the form of avoided illiquidity costs.
Proposition 1 shows that in Equilibrium II, bank loan supply is determined by certain marginal funding costs and uncertain marginal funding costs. Analogously to Equilibrium I for the case $\gamma > 0$, also in this equilibrium uncertain marginal funding costs increase in $\gamma$ and $\chi$. Hence, also in this equilibrium bank loan supply declines in the extent of uncertainty and transaction costs.

**Equilibrium III**

If the transaction costs parameter reaches the critical level $\bar{\gamma}$, banks with a liquidity deficit will no longer be willing to borrow their liquidity from the interbank market but prefer to use the central bank’s lending facility instead, i.e. $LF^* > 0$. The interbank market breaks down. Both, deficit banks as well as surplus banks, exclusively use the facilities to balance their liquidity needs at the second stage. However, also in this equilibrium banks borrow more liquidity from the central bank’s refinancing operations than they expect to need. As in Equilibrium II, there is a precautionary demand for liquidity leading to an aggregate excess liquidity in the amount of $RO^* - cL^*$, i.e. $I^* > 0$. As the precautionary demand for liquidity has to be placed in the deposit facility, it follows that $DF^* > LF^*$. This Equilibrium III is captured by (13).

Analogously to Equilibrium II, rewriting (8) leads to

$$i^{RO} - i^{DF} G(I^*) = i^{LF} \left[ 1 - G(I^*) \right],$$

which shows expected marginal costs and revenues of holding precautionary liquidity in Equilibrium III. Analogously to Equilibrium II, the left hand side of (16) represents expected marginal costs and the right hand side expected marginal revenues. Note that due to $i^{DF} + 2\gamma \leq i^{LF}$ it follows that $I^*$, and thus the precautionary demand for liquidity, in Equilibrium III is larger than in Equilibrium II.

Due to the breakdown of the interbank market, interbank market transaction costs obviously do not influence uncertain marginal funding costs and therefore, neither bank loan

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20Due to $G(0) < 0.5$ and $i^{LF} - i^{RO} \geq i^{RO} - i^{DF}$ expected marginal revenues of borrowing from the refinancing operations will exceed marginal costs if $RO = cL$, i.e. if $\bar{I} = 0$ (see Lemma 1).
supply. However, analogously to Equilibrium II an increase in the extent of uncertainty $\chi$ reduces bank loan supply $L^*$ as revealed by (14).

**Further Comments**

In Proposition 1 we assume that $G(0) < 0.5$ and that $i_{LF} - i_{RO} \geq i_{RO} - i_{DF}$. These assumptions imply that the equilibrium is characterized by $RO^* \geq cL^*$, which is the situation observed in the euro area. However, for the sake of completeness, we will briefly comment on the possible equilibrium $RO^* < cL^*$. This equilibrium will emerge if the distribution of $T$ becomes sufficiently right-skewed or if the interest corridor becomes sufficiently asymmetric with $i_{LF} - i_{RO} < i_{RO} - i_{DF}$.

We start with the importance of the distribution of $T$. Suppose that $T$ is distributed symmetrically around zero and that the interest rates on the facilities form a symmetric corridor around the policy rate. Then, the probability of facing a net deposit outflow equals the probability of facing a net deposit inflow. In this case, only $RO_b = cL_b$ implies that (8) is fulfilled. Transaction costs play no role as they increase marginal revenues of borrowing from the refinancing operations in the case of a liquidity deficit and decrease them in the case of a liquidity surplus, and given the symmetric distribution of $T$, both scenarios occur with the same probability for $RO_b = cL_b$. Now let us assume that the distribution of $T$ becomes right-skewed. For $RO_b = cL_b$, the probability of facing a liquidity surplus increases. In this case, transaction costs imply that expected marginal revenues of borrowing from the refinancing operations decrease. Accordingly, banks are incentivized to borrow less from the refinancing operations. Analogously to the case of a left-skewed distribution of $T$ this results in an increase in the interbank rate to balance marginal costs and expected marginal revenues of borrowing from the refinancing operations. However, if the interbank rate reaches its upper bound so that it cannot increase further, banks start to borrow less from the refinancing operations and $RO^* < cL^*$.

In order to highlight the importance of the asymmetry of the interest corridor let us assume that $T$ is distributed symmetrically around zero. If the asymmetry of the interest corridor is then given by $i_{LF} - i_{RO} < i_{RO} - i_{DF}$ and if $i_{LF} - i_{RO} < \gamma < i_{RO} - i_{DF}$,
optimal bank behavior will imply $RO_b^{opt} < cL_b^{opt}$. Facing relatively high transaction costs $(\ell - i^R < \gamma)$, banks with a liquidity deficit only accept an interbank rate below $i^R$. Otherwise, they prefer to use the lending facility. However, an interbank rate below $i^R$ implies for $RO_b = cL_b$ expected marginal revenues of borrowing from the refinancing operations to be lower than marginal costs. Therefore, banks are incentivized to borrow less from the refinancing operations. Generally, this leads to an increase in the interbank rate balancing expected marginal revenues and marginal costs again. However, such an adjustment is not possible as in the case of a liquidity deficit a bank would not accept a higher interbank rate. Therefore, all banks actually start to borrow less from the refinancing operations to balance expected marginal revenues and marginal costs so that $RO^* < cL^*$.

7 Monetary Policy and Bank Loan Supply

7.1 Banks’ Expected Marginal Funding Costs

Monetary policy influences bank loan supply by changing banks’ expected marginal funding costs. Therefore, we look at these costs in more detail before analyzing the impact

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21 For $RO_b^{opt} = cL_b^{opt}$ we get that $\bar{t}_b = 0$ as equation (3) shows. This means for a symmetric distribution of $T$ that $G(\bar{t}_b = 0) = 0.5$, so that expected marginal revenues of borrowing from the refinancing operations are equal to the interbank rate as revealed by the right hand side of equation (8). As marginal costs of borrowing from the refinancing operations are given by $i^R$, $i^{BM} < i^R$ implies expected marginal revenues to be lower than marginal costs.
of different monetary policy impulses. The following equations show expected marginal funding costs for each equilibrium.22

Equilibrium I:  
\[ ci^{RO} + (1 - c) \chi E \left[ T | t_b < t^* \right] G \left( t^* \right) \left( i^{IBM} - \gamma \right) \]
\[ + (1 - c) \chi E \left[ T | t_b > t^* \right] \left[ 1 - G \left( t^* \right) \right] \left( i^{IBM} + \gamma \right) = ci^{RO} + (1 - c) \chi E \left[ T | t_b > t^* \right] \left[ 1 - G \left( t^* \right) \right] 2\gamma, \] (17)

Equilibrium II:  
\[ ci^{RO} + (1 - c) \chi E \left[ T | t_b < t^* \right] G \left( t^* \right) i^{DF} \]
\[ + (1 - c) \chi E \left[ T | t_b > t^* \right] \left[ 1 - G \left( t^* \right) \right] \left( i^{DF} + 2\gamma \right) = ci^{RO} + (1 - c) \chi E \left[ T | t_b > t^* \right] \left[ 1 - G \left( t^* \right) \right] 2\gamma, \] (18)

Equilibrium III:  
\[ ci^{RO} + (1 - c) \chi E \left[ T | t_b < t^* \right] G \left( t^* \right) i^{DF} \]
\[ + (1 - c) \chi E \left[ T | t_b > t^* \right] \left[ 1 - G \left( t^* \right) \right] i^{LF} \]
\[ = ci^{RO} + (1 - c) \chi E \left[ T | t_b > t^* \right] \left[ 1 - G \left( t^* \right) \right] \left( i^{LF} - i^{DF} \right). \] (19)

If banks increase lending by one unit, they will face additional certain liquidity needs \( c \) due to cash withdrawals and additional uncertain liquidity needs \( (1 - c) \chi t_b \) due to deposit transfers. The first line of each of the equations (17) to (19) shows certain marginal funding costs due to cash withdrawals. The second and third line represent uncertain marginal funding costs resulting from deposit transfers. The second line reveals expected marginal revenues that accrue in the case of a liquidity surplus. The third line reflects expected marginal costs in case a liquidity deficit occurs. In Equilibrium I and II, \( i^{IBM} \) and \( i^{DF} \) respectively, imply interest costs in the case of a deficit and interest revenues in the case of a surplus. This means, that both rates are irrelevant for the respective expected marginal funding costs as the last line of (17) and (18) shows.

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22Note that with respect to the last line of each of the equations (17) to (19) we use the fact that 
\[ (1 - c) \chi E \left[ T | t_b < t^* \right] G \left( t^* \right) = -(1 - c) \chi E \left[ T | t_b > t^* \right] \left[ 1 - G \left( t^* \right) \right] \] due to \( E[T] = 0. \)
This last line of each of the equations (17) to (19) reveals that in all equilibria, expected marginal funding costs consist of certain marginal funding costs which are given by the currency ratio \(c\) and the policy rate \(i^{RO}\), and uncertain marginal funding costs which are determined by the expected liquidity deficit \((1-c)\chi E[T|t_b > \bar{t}^*]\), the costs \(\gamma\) or \(i^{LF} - i^{DF}\), and the probability of facing a deficit \(1 - G(\bar{t}^*)\). The effectiveness of monetary policy on bank loan supply depends on its influence on these three components of banks’ expected marginal funding costs. Against this background, we will now analyze the effectiveness of different monetary policy impulses.

7.2 Likewise Change in All Interest Rates

From the previous section it follows that the central bank has several alternatives to impose an effect on bank loan supply. Starting with the alternative which has been mostly used by the ECB, a likewise change in all interest rates, we obtain

**Proposition 2:** A likewise change in all central bank’s interest rates, i.e. \(di^{RO} = di^{DF} = di^{LF}\), implies for Equilibrium \(j = \{I, II, III\}\)

\[
\frac{\partial L^{j^*}}{\partial i^{RO}} = \frac{c}{\lambda} < 0 \quad \text{and} \quad \frac{\partial^2 L^{j^*}}{\partial i^{RO} \partial \chi} = \frac{\partial^2 L^{j^*}}{\partial i^{RO} \partial \gamma} = 0 \quad \forall \quad j.
\]

**Proof:** See appendix.

The proposition reveals that the impact of a likewise change in all central bank interest rates on bank loan supply is the same in all equilibria. A decrease in all interest rates has an expansionary effect on bank loan supply. The strength of this effect increases in \(c\). The extent of uncertainty about deposit transfers \(\chi\) as well as the transaction costs parameter \(\gamma\) play no role for the effectiveness of this monetary policy impulse. The reason is that in all equilibria, the decrease in \(i^{RO}\) implies certain marginal funding costs to become lower, whereas uncertain marginal funding costs do not change. In the following, we will comment on the latter.

In Equilibrium I, the decrease in the rates on the facilities is not relevant for banks’ loan supply decision as the facilities are not used and as banks do not expect to use them. The decrease in \(i^{RO}\) implies a likewise decrease in the interbank rate \(i^{IBM^*}\). This means that banks have no incentive to change their borrowing behavior with respect to
the refinancing operations. They still cover only their certain liquidity needs by borrowing from this liquidity source, i.e. $\bar{t}^{\ast}$ remains zero. Therefore, uncertain marginal costs, given by the second term on the right hand side of (17), do not change, as none of the three components (the expected liquidity deficit, the relevant costs, the probability of facing a deficit) change.

Although in Equilibrium II and III the facilities are used, also in these equilibria uncertain marginal funding costs will not change if the central bank decreases all its interest rates to the same extent. The decrease in $i^{RO}$ results in a decrease in marginal costs of borrowing from the refinancing operations whereas lowering the rates on the facilities implies a likewise decrease in expected marginal revenues. Therefore, banks have no incentive to adjust their borrowing behavior with respect to the refinancing operations. They do not alter their precautionary holdings of liquidity relative to their lending to the non-banking sector. Consequently, $\bar{t}^{\ast} \geq 0$ remains unchanged so that neither the probability of facing a deficit, nor the expected liquidity deficit varies. As the relevant costs do not change either, the monetary policy impulse does not affect banks’ uncertain marginal funding costs as described by the second term on the right hand side of (18) and (19).

In June 2014, for the first time the ECB imposed a negative rate on its deposit facility. For the effect of a change in all central bank interest rates on bank loan supply, the sign of the deposit rate is irrelevant. The deposit rate influences bank loan supply as it determines expected marginal costs and revenues of holding precautionary liquidity and, therefore, the probability of facing a liquidity deficit and, therefore again, uncertain marginal funding costs. However, for the influence on expected marginal revenues/costs of holding precautionary liquidity, the sign of the deposit rate plays no role. Crucial is only the spread between $i^{RO}$ and $i^{DF}$ as (15) and (16) show.

7.3 Change in the Width of the Interest Corridor

Alternatively of changing all interest rates to the same extent, the central bank may change the width of the interest corridor around its policy rate. Until September/October 2008, when the recent financial crisis reached its peak, the ECB did not make use of this policy.
instrument. However, since then it changed the width several times. For this case, we obtain

**Proposition 3:** Suppose, we have a symmetric interest corridor, i.e. \( i^{RO} = \frac{i^{DF} + i^{LF}}{2} \). A change in the width of the interest corridor, i.e. \( di^{LF} = -di^{DF} \) and \( di^{RO} = 0 \), implies

\[
\frac{\partial L^{I*}}{\partial (i^{LF} - i^{DF})} = 0,
\]
\[
\frac{\partial L^{II*}}{\partial (i^{LF} - i^{DF})} = -\frac{(1-c)\chi T^*}{2\lambda} < 0,
\]
\[
\frac{\partial L^{III*}}{\partial (i^{LF} - i^{DF})} = -\frac{(1-c)\chi}{\lambda} E[T|t_b > T^*] [1 - G(T^*)] < 0,
\]

with

\[
\frac{\partial^2 L^{I*}}{\partial (i^{LF} - i^{DF}) \partial \chi} = 0, \quad \frac{\partial^2 L^{II*}}{\partial (i^{LF} - i^{DF}) \partial \chi} < 0, \quad \frac{\partial^2 L^{III*}}{\partial (i^{LF} - i^{DF}) \partial \chi} < 0,
\]
\[
\frac{\partial^2 L^{I*}}{\partial (i^{LF} - i^{DF}) \partial \gamma} = 0, \quad \frac{\partial^2 L^{II*}}{\partial (i^{LF} - i^{DF}) \partial \gamma} < 0, \quad \frac{\partial^2 L^{III*}}{\partial (i^{LF} - i^{DF}) \partial \gamma} = 0.
\]

**Proof:** See appendix.

Proposition 3 shows that by changing the width of the interest corridor, the central bank may influence bank loan supply without changing its policy rate \( i^{RO} \). As \( i^{RO} \) remains unchanged, certain marginal funding costs do not alter. A potential effect of this monetary policy impulse on bank loan supply must thus be due to changed uncertain marginal funding costs. As in Equilibrium I banks never use the facilities to balance their uncertain liquidity needs, changing the width of the corridor has no effect on their loan supply decision.

In Equilibrium II, the change in \( i^{LF} \) is irrelevant as the lending facility is not (expected to be) used. However, the decrease in \( i^{DF} \) implies that banks reduce their precautionary holdings of liquidity as it becomes relatively expensive, see (15). These reduced precautionary liquidity holdings lead to an increase in the probability of facing a liquidity deficit.

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23From April 1999 until October 2008 the width of the corridor was constantly 200 basis points. In October 2008 the ECB reduced the width to 100 and 50 basis points, but increased it again to 200 basis points in January 2009. However, since then, the ECB reduced the width in several steps to currently 50 basis points.
$1 - G(T^*)$ so that uncertain marginal funding costs increase. This has a contractionary effect on bank loan supply.\footnote{Formally, the reduced holdings of precautionary liquidity are reflected by a decrease in $\bar{t}^*$. Note that this decrease does not only imply that $1 - G(T^*)$ increases but also that $E[T|t_b > T^*]$ decreases, i.e.

$$\frac{\partial E[T|t_b > T^*]}{\partial T^*} = \frac{\int_{t_b}^{\max} r(t_b - t^*)n(t_b) dt_b}{\left[\int_{t_b}^{\max} g(t_b) dt_b \right]^2} > 0.$$  However, the overall effect of a decrease in $T^*$ on uncertain marginal funding costs is positive:

$$\frac{\partial E[T|t_b > T^*][1 - G(T^*)]}{\partial T^*} = -(1 - c)\chi^2 g(T^*) \bar{T}^* < 0.$$}

It follows from (18) that the effect of the increased probability of facing a deficit on expected marginal funding costs is the stronger the higher $\gamma$ is. However, Proposition 3 shows that $\gamma$ has no direct effect on the strength of the impact of a change in the width on bank loan supply. The reason is that although the costs increase in $\gamma$ the increase in $T^*$, and therefore, the decrease in the probability of facing a liquidity deficit, is smaller for higher $\gamma$.\footnote{It follows from (15) that the probability of facing a liquidity deficit increases in $\gamma$, as banks will hold more precautionary liquidity if transaction costs are higher.} These two effects compensate each other. However, as the expected deficit increases in $\gamma$,\footnote{The expected deficit increases in $\gamma$, as $\bar{t}^*$ increases in $\gamma$ (see Lemma 1).} transaction costs indirectly influence the strength of the impact of this monetary policy impulse on bank loan supply. Consequently, widening the interest corridor imposes a more pronounced contractionary effect on bank loan supply the higher banks’ transaction costs and the extent of uncertainty are.

In Equilibrium III, banks do not adjust their precautionary liquidity holdings relative to their loan supply, i.e. $\bar{t}^*$ remains unchanged as the symmetry of the interest corridor implies that for all $i^{DF}$ and $i^{LF}$ forming a symmetric corridor around $i^{RO}$, $G(T^*) = 1 - G(T^*) = 0.5$. Only in this case, marginal costs of borrowing from the refinancing operations equal expected marginal revenues. Consequently, expected marginal funding costs will only change because the costs $i^{LF} - i^{DF}$ change as revealed by (19). In case the width of the corridor increases, uncertain marginal funding costs increase resulting in a contractionary effect on bank loan supply. The increase in uncertain marginal funding costs and, therefore, the impact on bank loan supply is the stronger, the higher the expected deficit and the probability of facing a liquidity deficit are. As the expected deficit increases in $\chi$, also the effect of this monetary policy impulse on bank loan supply increases in the extent of uncertainty. Note that the sign of the deposit rate is again irrelvant for the monetary policy impulse.
7.4 Change in the Asymmetry of the Interest Corridor

From April 1999 until November 2013, the rates on the facilities set by the ECB have formed a symmetric corridor around the policy rate $i^{RO}$. However, in November 2013, only the lending rate and the policy rate were reduced. The deposit rate remained unchanged to avoid a negative deposit rate. Consequently, the corridor became asymmetric. However, in June 2014, the ECB set a negative deposit rate. Since then, the corridor has been again symmetric. This section shows how the implementation/change of an asymmetric corridor affects bank loan supply. There are several ways to implement/change this asymmetry. We will have a closer look at two possibilities: a change in the policy rate $i^{RO}$ without changing the rates on the facilities and a likewise change in the rates on the facilities $i^{LF}$ and $i^{DF}$ without changing the policy rate.

Suppose the central bank only changes its policy rate $i^{RO}$. Then, we obtain

Proposition 4: A change in the policy rate implies for Equilibrium $j$

$$\frac{\partial L^{j*}}{\partial i^{RO}} = -\frac{1}{\lambda} \left[ c + (1 - c)\chi^{f} \right] < 0 \; \forall \; j \text{ with } \frac{\partial L^{III*}}{\partial i^{RO}} < \frac{\partial L^{II*}}{\partial i^{RO}} < \frac{\partial L^{I*}}{\partial i^{RO}} < 0.$$

Furthermore, we get

$$\frac{\partial^2 L^{III*}}{\partial i^{RO} \partial \chi} < \frac{\partial^2 L^{II*}}{\partial i^{RO} \partial \chi} < \frac{\partial^2 L^{I*}}{\partial i^{RO} \partial \gamma} = 0,$$

$$\frac{\partial^2 L^{III*}}{\partial i^{RO} \partial \gamma} = 0, \quad \frac{\partial^2 L^{II*}}{\partial i^{RO} \partial \gamma} < 0, \quad \frac{\partial^2 L^{I*}}{\partial i^{RO} \partial \gamma} = 0.$$

Proof: See appendix.

Proposition 4 reveals that in all equilibria, the central bank affects bank loan supply by changing only its policy rate $i^{RO}$. However, the impact of this monetary policy impulse on bank loan supply is stronger in Equilibrium II and III than in Equilibrium I. To comment on this aspect let us assume that the central bank decreases $i^{RO}$.

In Equilibrium I, a decrease in $i^{RO}$ reduces marginal costs of borrowing from the refi-nancing operations below expected marginal revenues. However, due to the perfectly functioning interbank market this leads to a likewise decrease in the interbank rate $i^{BM*}$.
Accordingly, banks do not change their borrowing behavior from the central bank’s refinancing operations. They still cover exactly their certain liquidity needs $cL^*$ from this liquidity source, i.e. $\tilde{t}^*$ remains zero. Therefore, none of the three components determining uncertain marginal costs as described by the last line of (19) is affected by this monetary policy impulse. However, the decrease in certain funding costs increases bank loan supply.

In contrast, in Equilibrium II and III both certain and uncertain marginal funding costs will decrease if the central bank solely lowers its policy rate $i^{RO}$. The reason is that in these equilibria the interbank market does not function (properly). If the central bank lowers $i^{RO}$, a decrease in the interbank rate to balance marginal costs and expected marginal revenues of borrowing from the refinancing operations will not be feasible. Either the interbank market rate is at its lower bound (Equilibrium II) or an interbank market does not exist due to too high transaction costs (Equilibrium III). Accordingly, expected marginal costs of precautionary liquidity holdings become lower than expected marginal revenues as shown by equation (15) and (16). Banks thus increase their precautionary liquidity holdings by borrowing more liquidity from the central bank’s refinancing operations and $\tilde{t}^*$ increases. Higher holdings of precautionary liquidity imply that the probability of facing a liquidity deficit decreases, which balances marginal costs and expected marginal revenues of borrowing from the refinancing operations.

The decrease in the probability of facing a liquidity deficit is responsible for banks’ reduced uncertain marginal funding costs as revealed by (18) and (19). Consequently, there is not only a decrease in certain but also in uncertain marginal funding costs implying an additional positive effect on bank loan supply. The effect of the decreasing probability on uncertain marginal funding costs is the stronger, the higher the expected deficit and the higher the relevant costs are. However, analogously to the case described in Section 7.3, the transaction costs parameter $\gamma$ only has an indirect impact on bank loan supply in Equilibrium II. In Equilibrium III, $\tilde{t}^*$ is larger than in Equilibrium II so that the expected liquidity deficit is also higher. This means that in Equilibrium III the effect of a decreasing $i^{RO}$ on bank loan supply is stronger than in Equilibrium II.
Another possibility to implement/change the asymmetry of the interest corridor is a likewise change in the rates on the central bank’s facilities. For this monetary policy impulse we obtain

**Proposition 5:** A change in the interest corridor in the form of \( d_{i}^{LF} = d_{i}^{DF} \) and \( d_{i}^{RO} = 0 \), implies

\[
\frac{\partial L^{I^*}}{\partial i^{DF}} = 0,
\]

\[
\frac{\partial L^{I^*}}{\partial i^{DF}} = \frac{(1 - c) \chi}{2 \lambda} \tau^* > 0 \quad \text{for} \quad j = II, III,
\]

with

\[
\frac{\partial L^{III^*}}{\partial i^{DF}} > \frac{\partial L^{II^*}}{\partial i^{DF}} > \frac{\partial L^{I^*}}{\partial i^{DF}} = 0.
\]

Furthermore, we get

\[
\frac{\partial^2 L^{III^*}}{\partial i^{DF} \partial \chi} > 0, \quad \frac{\partial^2 L^{II^*}}{\partial i^{DF} \partial \chi} > 0, \quad \frac{\partial^2 L^{I^*}}{\partial i^{DF} \partial \chi} = 0,
\]

\[
\frac{\partial^2 L^{III^*}}{\partial i^{DF} \partial \gamma} = 0, \quad \frac{\partial^2 L^{II^*}}{\partial i^{DF} \partial \gamma} > 0, \quad \frac{\partial^2 L^{I^*}}{\partial i^{DF} \partial \gamma} = 0.
\]

**Proof:** See appendix.

Let us assume that the central bank decreases the rates on its facilities. As \( i^{RO} \) remains unchanged, in neither equilibrium certain marginal funding costs are affected by this monetary policy impulse. As in Equilibrium I the facilities are not used, a change in their rates does not affect banks’ loan supply decision at all. However, in Equilibrium II and III a likewise decrease in the rates on the facilities increases banks’ uncertain marginal funding costs so that bank loan supply decreases. The decrease in the interest rates implies that expected marginal costs of precautionary liquidity holdings increase whereas expected marginal revenues decrease, as (15) and (16) show. Consequently, banks reduce their holdings of precautionary liquidity. This leads to an increase in the probability of facing a liquidity deficit, and therefore, to an increase in uncertain marginal funding costs so that bank loan supply decreases. Analogously to the previously discussed case, the strength of this monetary impulse increases in the expected liquidity deficit \((1 - c) \chi E[T|t_b > \tau^*] \). As
### Table 1: Impact of Different Monetary Policy Impulses

The expected deficit is larger in Equilibrium III than in Equilibrium II, the contractionary effect on bank loan supply is stronger in Equilibrium III. Furthermore, the strength of this impact increases in transaction costs (Equilibrium II) and the extent of uncertainty (Equilibrium II and III). Again, the sign of the interest rates has no impact.

The monetary policy impulses discussed in this section differ in their impact on marginal funding costs, bank loan supply, and banks’ precautionary demand for liquidity. Table 1 presents an overview of these effects.

#### 7.5 Collateralization and Minimum Reserves

We have neglected two elements of the ECB’s operational framework: the collateralization of central bank credits and minimum reserve requirements. Considering these elements is not crucial for our analysis. They only change our results quantitatively.

If we considered the collateralization of central bank credits, banks would face opportunity costs of holding collateral. As a result, expected marginal funding costs would
Bank loan supply would be lower but our model results would not change qualitatively.

Reserve requirements lead to a structural liquidity deficit of the banking sector. In our model, such a deficit is already captured by considering cash withdrawals. Introducing reserve requirements would therefore simply increase the existing structural deficit. A main feature of the Eurosystem’s minimum reserve system is that banks can make use of averaging provision of required reserves during the reserve maintenance period. This allows banks to smooth out liquidity fluctuations. In our model, uncertain marginal funding costs, and thus banks’ expected marginal funding costs would decrease. Although this would have a positive effect on bank loan supply, the qualitative results of our model would not change.

8 Conclusions

The interbank market plays a crucial role for the implementation of monetary policy as it serves as the starting point of different transmission mechanisms. Based on a theoretical model, this paper analyzes in how far interbank market frictions in the form of transaction costs influence the effectiveness of monetary policy on bank loan supply and draws conclusions for monetary policy implementation.

We show that interbank market frictions are not an impediment for the effectiveness of monetary policy. On the contrary, sufficiently high interbank market transaction costs allow the central bank to reinforce the impact of its policy on banks’ marginal funding costs and therefore, on bank loan supply.

If the central bank changes all its interest rates (deposit rate, main refinancing rate, lending rate) to the same extent, it will only affect banks’ certain marginal funding costs. However, by changing the width or asymmetry of the interest corridor the central bank may also influence banks’ uncertain marginal funding costs. Crucial is that sufficiently high transaction costs imply that banks start to hold precautionary liquidity. By changing the interest corridor the central bank alters marginal costs and revenues of this liquidity and, therefore, how much precautionary liquidity banks actually hold which determines

\footnote{For a respective analysis see, for example, Neyer and Wiemers (2004) as well as Berentsen and Monnet (2008).}
their uncertain marginal funding costs. Consequently, for sufficiently large transaction costs the interest corridor can be used as an effective monetary policy instrument.

This allows us to draw some conclusions for the Eurosystem’s monetary policy. From October 2008 up to now (February 2015), the ECB has not only decreased its main refinancing rate from 4.75% to an historically low level of 0.05%, but it has also narrowed the corridor from 200 basis points to 50 basis points. The ECB has thus not only significantly reduced banks’ certain marginal funding costs but also their uncertain marginal funding costs. Although the interbank market is important for monetary policy implementation, its malfunctioning has thus not been an impediment for the effectiveness of monetary policy. On the contrary, frictions in the interbank market have allowed the ECB to impose an even stronger effect on banks’ funding costs. However, despite banks’ very low funding costs, bank lending in the euro area, especially in the periphery, has been declining. This indicates that there are structural problems, as insufficient bank capital or the lack of competitive projects to be financed, being responsible for the missing stimulating effect of an extremely expansionary monetary policy on bank lending.

Appendix

Proof of Lemma 1

It follows from (5) and (7) that the first stage optimization problem of bank $b$ reads:

$$\max_{L_b, RO_b} E[\pi_b] = i^L_L - \frac{1}{2} \lambda L_b^2 - i^{RO} RO_b - \max \{i^{IBM} - \gamma, i^{DF}\} \int_{t_{min}}^{t_b} N_b g(t_b) \, dt_b - \min \{i^{IBM} + \gamma, i^{LF}\} \int_{t_b}^{t_{max}} N_b g(t_b) \, dt_b,$$

subject to (6) and (3). By applying the Leibniz rule and making use of the fact that $N_b = 0$ for $t_b = \bar{t}_b$, we obtain:

$$\frac{\partial E[\pi_b]}{\partial RO_b} = -i^{RO} - \max \{i^{IBM} - \gamma, i^{DF}\} \int_{t_{min}}^{t_b} \frac{\partial N_b}{\partial RO_b} g(t_b) \, dt_b$$

$$- \min \{i^{IBM} + \gamma, i^{LF}\} \int_{t_b}^{t_{max}} \frac{\partial N_b}{\partial RO_b} g(t_b) \, dt_b.$$
We can infer from (2) that $\frac{\partial N_b}{\partial RO_b} = -1$. Insertion of this in (22) and rewriting terms yields

$$\frac{\partial E[\pi_b]}{\partial RO_b} = -i^{RO} + \max \{i^{IBM} - \gamma, i^{DF}\} G(\bar{t}_b) + \min \{i^{IBM} + \gamma, i^{LF}\} [1 - G(\bar{t}_b)].$$  \hspace{1cm} (23)

Note that $\frac{\partial E[\pi_b]}{\partial RO_b}$ decreases in $G(\bar{t}_b) \in [0, 1]$, which in turn (weakly) increases in $\bar{t}_b$. Moreover, we know from (3) that

- $\bar{t}_b$ increases in $RO_b$, so that $\frac{\partial E[\pi_b]}{\partial RO_b}$ (weakly) decreases in $RO_b$,
- and from the restriction $RO_b \geq 0$ that $\bar{t}_b$ is restricted to $\bar{t}_b \geq -\frac{c}{1-c} \chi =: \hat{t}$.

Denoting optima by the superscript $\text{opt}$, we can distinguish three cases:

1. If $i^{IBM} > i^{RO} + \gamma$, then $\frac{\partial E[\pi_b]}{\partial RO_b} > 0$ for all $G(\bar{t}_b)$. Therefore, we obtain $\bar{t}_b^{\text{opt}} = \infty$. In conjunction with (3), this yields $RO_b^{\text{opt}} = \infty$.

2. If $i^{IBM} \in [i^{RO} - \gamma, i^{RO} + \gamma]$, then $\frac{\partial E[\pi_b]}{\partial RO_b} = 0$ only if $\bar{t}_b = \bar{t}_b^{\text{opt}}$, where $\bar{t}_b^{\text{opt}}$ is implicitly defined by (8). In conjunction with (3) and the restriction $RO_b \geq 0$, this yields $RO_b^{\text{opt}} = \max \{0, \bar{t}_b^{\text{opt}} (1-c) \chi L_b^{\text{opt}} + c L_b^{\text{opt}}\}$, which brings us to two subcases.

   - If $\bar{t}_b^{\text{opt}} > \hat{t}$ and thus $G(\bar{t}_b^{\text{opt}}) > G(\hat{t})$, then $RO_b^{\text{opt}} = \bar{t}_b^{\text{opt}} (1-c) \chi L_b^{\text{opt}} + c L_b^{\text{opt}} > 0$.
   - If $\bar{t}_b^{\text{opt}} \leq \hat{t}$ and thus $G(\bar{t}_b^{\text{opt}}) \leq G(\hat{t})$, then $RO_b^{\text{opt}} = 0$.

3. If $i^{IBM} < i^{RO} - \gamma$, then $\frac{\partial E[\pi_b]}{\partial RO_b} < 0$ for all $G(\bar{t}_b)$. Therefore, we obtain $\bar{t}_b = -\infty$.

   In conjunction with (3) and the restriction $RO_b \geq 0$, this yields $RO_b^{\text{opt}} = 0$.

Consequently, we have shown that only if $i^{IBM^*} \in [i^{RO} - \gamma, i^{RO} + \gamma]$ a bank’s optimal borrowing from the refinancing operations is described by the first order condition given by equation (8). Note that we restrict the analysis in the paper to $\bar{t}_b^{\text{opt}} \geq \hat{t} := -\frac{c}{(1-c) \chi}$.

**Proof of Lemma 2**

From (3) and the restriction $RO_b \geq 0$ it follows that $\bar{t}_b$ is restricted to $\bar{t}_b \geq -\frac{c}{(1-c) \chi} =: \hat{t}$.

By applying the Leibniz rule on (21) and making use of the facts that $N_b = 0$ for $t_b = \bar{t}_b$
and that optimal borrowing from the refinancing operations implies \( \bar{t}_b = \max \left\{ t_{opt}^b, \tilde{t} \right\} \), we obtain:

\[
\frac{\partial E[\pi]}{\partial L_b} = i^L - \lambda L_b - \max \left\{ i^{IBM} - \gamma, i^{DF} \right\} \int_{t_{min}}^{t_{max}} \frac{\partial N_b}{\partial L_b} g(t_b) dt_b - \min \left\{ i^{IBM} + \gamma, i^{LF} \right\} \int_{\max \left\{ t_{opt}^b, \tilde{t} \right\}}^{t_{min}} \frac{\partial N_b}{\partial L_b} g(t_b) dt_b - \max \left\{ i^{IBM} - \gamma, i^{DF} \right\} \int_{t_{min}}^{\max \left\{ t_{opt}^b, \tilde{t} \right\}} \frac{\partial N_b}{\partial L_b} g(t_b) dt_b,
\]

\( (24) \)

We can infer from (2) and the envelope theorem that \( \frac{\partial N_b}{\partial L_b} = c + (1 - c) \chi t_b \) and \( \frac{\partial E[\pi]}{\partial RO_{opt}^b} = 0 \). Insertion of this in (24) and rewriting terms yields

\[
\frac{\partial E[\pi]}{\partial L_b} = i^L - \lambda L_b - (1 - c) \chi \max \left\{ i^{IBM} - \gamma, i^{DF} \right\} \int_{t_{min}}^{t_{max}} t_b g(t_b) dt_b - (1 - c) \chi \min \left\{ i^{IBM} + \gamma, i^{LF} \right\} G \left( \max \left\{ t_{opt}^b, \tilde{t} \right\} \right) - c \max \left\{ i^{IBM} - \gamma, i^{DF} \right\} \left( 1 - G \left( \max \left\{ t_{opt}^b, \tilde{t} \right\} \right) \right).
\]

\( (25) \)

This brings us to two cases.

- If \( t_{opt}^b > \tilde{t} \) and thus \( G \left( t_{opt}^b \right) > G \left( \tilde{t} \right) \), then insertion of (8) in (25) implies that \( \frac{\partial E[\pi]}{\partial L_b} = 0 \) only if (9) is met.

- If \( t_{opt}^b \leq \tilde{t} \) and thus \( G \left( t_{opt}^b \right) \leq G \left( \tilde{t} \right) \), then \( \frac{\partial E[\pi]}{\partial L_b} = 0 \) only if

\[
i^L = \lambda L_{opt}^b + c \max \left\{ i^{IBM} - \gamma, i^{DF} \right\} G \left( \tilde{t} \right) + c \min \left\{ i^{IBM} + \gamma, i^{LF} \right\} \left[ 1 - G \left( \tilde{t} \right) \right] + (1 - c) \chi \max \left\{ i^{IBM} - \gamma, i^{DF} \right\} E \left[ T \mid t_b < \tilde{t} \right] - G \left( \tilde{t} \right) + (1 - c) \chi \min \left\{ i^{IBM} + \gamma, i^{LF} \right\} E \left[ T \mid t_b > \tilde{t} \right] \left[ 1 - G \left( \tilde{t} \right) \right].
\]

\( (26) \)

Note that we restrict the analysis in the paper to \( t_{opt}^b \geq \tilde{t} := \frac{c}{(1 - c) \chi} \).

**Proof of Proposition 1**

We distinguish between an active and an inactive interbank market to determine the interbank rate and bank aggregate borrowing and lending decisions in equilibrium.
Active Interbank Market ($\gamma \leq \bar{\gamma}$)

It is useful to distinguish between three cases:

1. Suppose that an equilibrium exists with $RO^* < cL^*$. Then, we have $i^{IBM*} = i^{LF} - \gamma$
while according to (3) $RO^* = \bar{t}^* (1 - c) \chi L^* + c L^* \geq 0$ implies $\bar{t} < 0$ and thus

$$G(\bar{t}) < G(0).$$

Insertion of $i^{IBM*}$ in (8) yields

$$G(\bar{t}) = \frac{i^{LF} - i^{RO}}{2\gamma} < G(0).$$

For all $\gamma \in [0, \bar{\gamma}]$ it follows due to $i^{LF} - i^{RO} \geq i^{RO} - i^{DF}$ that $\frac{i^{LF} - i^{RO}}{2\gamma} > 0.5$. As we assume that $G(0) < 0.5$, $RO^* < cL^*$ does not constitute an equilibrium.

2. Suppose that an equilibrium exists with $RO^* = cL^*$. Then, we have $i^{IBM*} \in [i^{DF} + \gamma, i^{LF} - \gamma]$, while according to (3) $RO^* = \bar{t}^* (1 - c) \chi L^* + c L^* > 0$ implies $\bar{t} = 0$ and thus

$$G(\bar{t}) = G(0).$$

Insertion of $G(\bar{t})$ in (8) yields

$$i^{IBM*} = i^{RO} - \gamma + 2\gamma G(0),$$

and thus $G(0) \in \left[\frac{i^{DF} + 2\gamma - i^{RO}}{2\gamma}, \frac{i^{LF} - i^{RO}}{2\gamma}\right]$. As $RO^* = cL^*$, there is no aggregate liquidity deficit at the second stage so that neither the lending nor the deposit facility is used.

3. Suppose that an equilibrium exists with $RO^* > cL^*$. Then, we have $i^{IBM*} = i^{DF} + \gamma$
while according to (3) $RO^* = \bar{t}^* (1 - c) \chi L^* + c L^* > 0$ implies $\bar{t} > 0$ and thus

$$G(\bar{t}) > G(0).$$
Insertion of \( i^{IBM^*} \) in (8) yields

\[
G (\tilde{t}) = \frac{i^{DF} + 2\gamma - i^{RO}}{2\gamma} > G (0) .
\]

As \( RO^* > cL^* \), banks have to place their aggregate liquidity surplus of the second stage in the deposit facility so that \( DF^* > 0 \).

**Inactive Interbank Market \((\gamma > \bar{\gamma})\)**

It is useful to distinguish between the same three cases as for an active interbank market:

1. Suppose that an equilibrium exists with \( RO^* < cL^* \). Then, we have \( i^{IBM^*} \in [i^{LF} - \gamma, i^{DF} + \gamma] \) while \( RO^* = \tilde{t}^* (1 - c) \chi L^* + cL^* > 0 \) implies \( \tilde{t} < 0 \) and thus

\[
G (\tilde{t}) < G (0) .
\]

Insertion of \( i^{IBM^*} \) in (8) yields

\[
G (\tilde{t}) = \frac{i^{LF} - i^{RO}}{\tilde{t}^{DF} - i^{DF}} < G (0) .
\]

It follows due to \( i^{LF} - i^{RO} \geq i^{RO} - i^{DF} \) that \( \frac{i^{LF} - i^{RO}}{\tilde{t}^{DF} - i^{DF}} > 0.5 \). As we assume that \( G(0) < 0.5 \), \( RO^* < cL^* \) does not constitute an equilibrium.

2. Suppose that an equilibrium exists with \( RO^* = cL^* \). Then, we have \( i^{IBM^*} \in [i^{LF} - \gamma, i^{DF} + \gamma] \) while \( RO^* = \tilde{t}^* (1 - c) \chi L^* + cL^* > 0 \) implies \( \tilde{t} = 0 \) and thus

\[
G (\tilde{t}) = G (0) .
\]

Insertion of \( i^{IBM^*} \) in (8) yields again

\[
G (\tilde{t}) = \frac{i^{LF} - i^{RO}}{\tilde{t}^{DF} - i^{DF}} = G (0) .
\]

Due to \( G(0) < 0.5 \), \( RO^* = cL^* \) does not constitute an equilibrium either.
3. Suppose that an equilibrium exists with \( RO^* > cL^* \). Then, we have \( i^{IBM^*} \in [i^{LF} - \gamma, i^{DF} + \gamma] \) while \( RO^* = \bar{r}^* (1 - c) \chi L^* + cL^* > 0 \) implies \( \bar{r} > 0 \) and thus

\[
G(\bar{r}) > G(0).
\]

Insertion of \( i^{IBM^*} \) in (8) yields again

\[
G(\bar{r}) = \frac{i^{LF} - i^{RO}}{i^{LF} - i^{DF}} > G(0).
\]

Banks with a liquidity deficit have to borrow from the lending facility, while banks with a liquidity surplus have to place their excess liquidity in the deposit facility. As \( RO^* > cL^* \), it follows that \( DF^* > LF^* > 0 \).

**Proof of Proposition 2**

We proof this proposition in two steps. First, we apply the total derivative to determine the impact of a likewise change in all interest rates. Afterwards, we derive the respective mixed partial derivative with respect to \( \chi \) and \( \gamma \).

1. Given \( di^{LF} = di^{RO} = di^{DF} \), applying the total derivative on (14) yields

\[
dL_i = \frac{1}{\lambda} \left[ -cdi^{RO} \right] - \left[ (1 - c)\chi dT - \frac{\partial}{\partial T} \int_{t_b}^{t_{max}} t_b g(t_b) dt_b \right] \xi,
\]

with

\[
\xi = \begin{cases} 
2\gamma & \text{if } \gamma \leq \bar{\gamma}, \\
i^{LF} - i^{DF} & \text{if } \gamma > \bar{\gamma}.
\end{cases}
\]

We derive from (8) the function

\[
F := \max \{i^{IBM} - \gamma, i^{DF}\} G(\bar{r}) + \min \{i^{IBM} + \gamma, i^{LF}\} \left[ 1 - G(\bar{r}) \right] - i^{RO} = 0.
\]

(28)
Applying the total derivative on (28) yields

\[ \frac{\partial G(t^*)}{\partial t^*} dt^* = 0 \quad \text{if} \quad j = I, \]
\[ d_i^{DF} - 2\gamma \frac{\partial G(t^*)}{\partial t^*} dt^* - d_i^{RO} = 0 \quad \text{if} \quad j = II, \]
\[ d_i^{LF} - (i^{DF} - i^{LF}) \frac{\partial G(t^*)}{\partial t^*} dt^* + (d_i^{DF} - d_i^{LF})G(t^*) - d_i^{RO} = 0 \quad \text{if} \quad j = III. \]

Due to \( d_i^{RO} = d_i^{DF} = d_i^{LF} \) it follows for all \( j \) that \( dt^* = 0 \) so that

\[ \frac{dL^j}{dt^{j*}} = -\frac{c}{\lambda}. \tag{29} \]

2. It follows directly from (29) that \( \frac{\partial^2 L^j}{\partial i^{j*} \partial \gamma} = \frac{\partial^2 L^j}{\partial i^{j*} \partial \chi} = 0 \) for all \( j \).

**Proof of Proposition 3**

We proof this proposition in the same two steps as the proof of Proposition 2.

1. Given \( d_i^{LF} = -d_i^{DF} \) and \( d_i^{RO} = 0 \), applying the total derivative on (14) yields

\[ dL^j = -\frac{2\gamma(1 - c)}{\lambda} \chi dt^* \frac{\partial}{\partial t^*} \int_{t^*}^{t_{max}} t_b g(t_b) dt_b \quad \text{if} \quad j = \{I, II\}, \]
\[ dL^j = -\frac{1 - c}{\lambda} \chi \left[ (i^{LF} - i^{DF}) dt^* \frac{\partial}{\partial t^*} \int_{t^*}^{t_{max}} t_b g(t_b) dt_b \right. \]
\[ \left. + 2d_i^{LF} \int_{t^*}^{t_{max}} t_b g(t_b) dt_b \right] \quad \text{if} \quad j = III. \]

Moreover, applying the total derivative on (28) yields

\[ \frac{\partial G(t^*)}{\partial t^*} dt^* = 0 \quad \text{if} \quad j = I, \]
\[ d_i^{DF} - 2\gamma \frac{\partial G(t^*)}{\partial t^*} dt^* = 0 \quad \text{if} \quad j = II. \]

As long as the interest corridor is symmetric, it follows for \( \gamma > \bar{\gamma} \) that \( G(t^*) = 0.5 \) so that

\[ dt^* = 0 \quad \text{if} \quad j = III. \]
Considering $\frac{\partial L^I}{\partial (i_{LF} - i_{DF})} = 0.5$ and $\frac{\partial L^D}{\partial (i_{LF} - i_{DF})} = -0.5$ it follows

\[
\frac{\partial L^I}{\partial (i_{LF} - i_{DF})} = \frac{dL^I}{di_{LF}} \frac{\partial i_{LF}}{\partial (i_{LF} - i_{DF})} = 0, \\
\frac{\partial L^D}{\partial (i_{LF} - i_{DF})} = \frac{dL^D}{di_{DF}} \frac{\partial i_{DF}}{\partial (i_{LF} - i_{DF})} = \frac{(1 - c) \chi}{2\lambda} \tilde{t}^* < 0, \\
\frac{\partial L^D}{\partial (i_{LF} - i_{DF})} = \frac{dL^D}{di_{LF}} \frac{\partial i_{LF}}{\partial (i_{LF} - i_{DF})} = \frac{(1 - c) \chi}{\lambda} E [T|t_b > \tilde{t}^*_b] [1 - G(\tilde{t}^*_b)] < 0.
\] (30)

2. Applying the implicit function theorem on (28) yields

\[
\frac{\partial \tilde{t}^*}{\partial \chi} = -\frac{\partial F}{\partial \chi} = 0 \ \forall \ \tilde{t}^*,
\] (33)

so that it follows directly from (30) to (32) that

\[
\frac{\partial^2 L^I}{\partial (i_{LF} - i_{DF}) \partial \chi} = 0, \\
\frac{\partial^2 L^D}{\partial (i_{LF} - i_{DF}) \partial \chi} = \frac{(1 - c) \chi}{2\lambda} \tilde{t}^* < 0, \\
\frac{\partial^2 L^D}{\partial (i_{LF} - i_{DF}) \partial \chi} = \frac{(1 - c) \chi}{\lambda} E [T|t_b > \tilde{t}^*_b] [1 - G(\tilde{t}^*_b)] < 0.
\] (32)

Applying the implicit function theorem on (28) yields

\[
\frac{\partial \tilde{\gamma}^*}{\partial \gamma} = -\frac{\partial F}{\partial \gamma} = \frac{G(\tilde{t}^*)}{\gamma g(\tilde{t}^*)} \quad \text{if} \ j = I, \\
\frac{\partial \tilde{\gamma}^*}{\partial \gamma} = -\frac{\partial F}{\partial \gamma} = \frac{1 - G(\tilde{t}^*)}{\gamma g(\tilde{t}^*)} \quad \text{if} \ j = II, \\
\frac{\partial \tilde{\gamma}^*}{\partial \gamma} = -\frac{\partial F}{\partial \gamma} = 0 \quad \text{if} \ j = III.
\] (34) (35) (36)

Making use of the result obtained in (34) to (36), it follows that

\[
\frac{\partial^2 L^I}{\partial (i_{LF} - i_{DF}) \partial \gamma} = 0 \ \text{for} \ j = I, III, \\
\frac{\partial^2 L^D}{\partial (i_{LF} - i_{DF}) \partial \gamma} = \frac{(1 - c) (1 - G(\tilde{t}^*))}{2\lambda} \gamma g(\tilde{t}^*) < 0.
\]
Proof of Proposition 4

We proof this proposition in the same two steps as the proof of Proposition 2.

1. Applying the Leibniz rule on (14), the first derivative w.r.t. $i^{RO}$ reads

$$\frac{\partial L^{j*}}{\partial i^{RO}} = -\frac{1}{\lambda} \left[ \frac{c - (1 - c)\chi}{\partial i^{RO}} t^* g(t^*) \xi \right] \forall j,$$

with $\xi$ given by (27). Applying the implicit function theorem on (28) yields

$$\frac{\partial t^*}{\partial i^{RO}} = \frac{-\partial F}{\partial i^{RO}} \frac{\partial F}{\partial t^*} = 0 \text{ if } j = I,$$
$$\frac{\partial t^*}{\partial i^{RO}} = \frac{-\partial F}{\partial i^{RO}} \frac{\partial F}{\partial t^*} = \frac{-1}{\xi g(t^*)} \text{ if } j = \{II, III\}.$$

As $t^* = 0$ for $j = I$, it follows for all $j$

$$\frac{\partial L^{j*}}{\partial i^{RO}} = -\frac{1}{\lambda} \left[ \frac{c + (1 - c)\chi}{\partial i^{RO}} t^* \right].$$

If $\gamma \leq \bar{\gamma}$, it follows that $2\gamma \leq i^{LF} - i^{DF}$. In Equilibrium II expected marginal revenues of borrowing from the refinancing operations are given by

$$i^{DF} G(t_b^{opt}) + (i^{DF} + 2\gamma) \left[ 1 - G(t_b^{opt}) \right], \quad (37)$$

while in Equilibrium III they read

$$i^{DF} G(t_b^{opt}) + i^{LF} \left[ 1 - G(t_b^{opt}) \right]. \quad (38)$$

Expected marginal costs are $i^{RO}$ in both equilibria. Comparing (37) and (38) shows that $G(t_b^{opt})^II < G(t_b^{opt})^III$ so that $t_b^{optIII} < t_b^{optII}$. Due to $L_b^{opt} = L^*$ and $RO_b^{opt} = RO^*$, it follows that $t^{II} < t^{III}$ and thus $\frac{\partial L^{II*}}{\partial i^{RO}} \leq \frac{\partial L^{VI*}}{\partial i^{RO}} < \frac{\partial L^{IV*}}{\partial i^{RO}} < 0$.

2. In order to determine the mixed partial derivative with respect to $\chi$, we make use of the result obtained in (33). It thus follows that

$$\frac{\partial^2 L^{j*}}{\partial i^{RO} \partial \chi} = -\frac{1 - c}{\lambda} t^*,$$
so that \( \frac{\partial^2 L_{III}^{*}}{\partial R \partial \chi} < 0 \) and \( \frac{\partial^2 L_{II}^{*}}{\partial R \partial \chi} = 0 \).

In order to determine the mixed partial derivative with respect to \( \gamma \), we make use of the results obtained in (34) to (36). It thus follows that

\[
\begin{align*}
\frac{\partial^2 L_{I}^{*}}{\partial R \partial \gamma} &= 0, \\
\frac{\partial^2 L_{II}^{*}}{\partial R \partial \gamma} &= -\frac{1}{\lambda} (1 - c) \frac{\partial \tilde{\tau}}{\partial \gamma} - \frac{1}{\lambda} (1 - c) \chi \left[ 1 - G(\tilde{\tau}) \right] < 0, \\
\frac{\partial^2 L_{III}^{*}}{\partial R \partial \gamma} &= 0.
\end{align*}
\]

**Proof of Proposition 5**

We proof this proposition in the same two steps as the proof of Proposition 2.

1. Given \( d_i LF = d_i DF \) and \( d_i RO = 0 \), applying the total derivative on (14) yields

\[
\begin{align*}
dL_j^{*} &= -\frac{2\gamma(1 - c)}{\lambda} \chi d\tilde{\tau} \frac{\partial}{\partial \tilde{\tau}} \int_{\tau}^{t_{\max}} t_b g(t_b) dt_b \quad \text{if} \quad j = \{I, II\}, \\
dL_j^{*} &= -\frac{1 - c}{\lambda} \chi (iLF - iDF) d\tilde{\tau} \frac{\partial}{\partial \tilde{\tau}} \int_{\tau}^{t_{\max}} t_b g(t_b) dt_b \quad \text{if} \quad j = III.
\end{align*}
\]

Moreover, applying the total derivative on (28) yields

\[
\begin{align*}
\frac{\partial G(\tilde{\tau})}{\partial \tilde{\tau}} d\tilde{\tau} &= 0 \quad \text{if} \quad j = I, \\
d_i DF - 2\gamma \frac{\partial G(\tilde{\tau})}{\partial \tilde{\tau}} d\tilde{\tau} &= 0 \quad \text{if} \quad j = II, \\
d_i LF - (iLF - iDF) \frac{\partial G(\tilde{\tau})}{\partial \tilde{\tau}} d\tilde{\tau} &= 0 \quad \text{if} \quad j = III,
\end{align*}
\]

so that

\[
\begin{align*}
\frac{\partial L_{I}^{*}}{\partial iDF} &= 0, \\
\frac{\partial L_{j}^{*}}{\partial iDF} &= \frac{(1 - c)\chi}{\lambda} t^{*} > 0 \quad \text{for} \quad j = II, III.
\end{align*}
\]
2. Making use of the result obtained in (33), it follows directly from (39) and (40) that
\[
\frac{\partial^2 L^i}{\partial iDF \partial \chi} = 0,
\]
\[
\frac{\partial^2 L^j}{\partial iDF \partial \chi} = \frac{(1 - c) t^*}{\lambda} > 0 \text{ for } j = \text{II, III.}
\]

Making use of the result obtained in (35) and (36), it follows that
\[
\frac{\partial^2 L^j}{\partial iDF \partial \gamma} = 0 \text{ for } j = \text{I, III,}
\]
\[
\frac{\partial^2 L^\text{II}}{\partial iDF \partial \gamma} = \frac{(1 - c)}{\lambda} \frac{1 - G(t^*)}{\gamma g(t^*)} > 0.
\]

Bibliography


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